Xia Jianguo and Qin Hourong

Abstract

$r_0, r_1, \cdots, r_{a-1}$ be the least nonnegative residues of 0, b, $2b$, $\cdots$, $(a-1)b$ modulus a. In this note, we give several recurrence formulas for the number of pairs $\{i, j\}$ with $(i-j)(r_i - r_j) < 0$. These formulas together with Zolotareff's lemma give a proof of the Law of Reciprocity for Legendre symbol. Furthermore, we prove that if a is a positive odd integer and b an integer with $(a, b) = 1$, then the permutation $r_0, r_1, \cdots, r_{a-1}$ is even or odd according as the value of Jacobi symbol is 1 or -1. This gives an arithmetic meaning of Jacobi symbol.