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Bijective proofs for Fibonacci identities related to Zeckendorf's theorem,
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Abstract

In *Proofs that Really Count*, Benjamin and Quinn wrote that there were no known bijective proofs for certain identities that give instances of Zeckendorf's Theorem, for example, $5f_n = f_{n+3} + f_{n-1} + f_{n-4}$, where $n \geq 4$ and where f_k is the k-th Fibonacci number (there are analogous identities for ℓf_n for every positive integer ℓ). In this paper, we provide bijective proofs for $5f_n = f_{n+3} + f_{n-1} + f_{n-4}$ and the seven other examples of such identities listed in *Proofs that Really Count*. We interpret f_k as the cardinality of the set $\mathcal{F}_k$ consisting of all ordered lists of 1's and 2's whose sum is k. We then demonstrate bijections that prove the eight identities listed in *Proofs that Really Count*; for example, to prove $5f_n = f_{n+3} + f_{n-1} + f_{n-4}$, we give a bijection between $\{1, 2, 3, 4, 5\} \times \mathcal{F}_n$ and $\mathcal{F}_{n+3} \cup \mathcal{F}_{n-1} \cup \mathcal{F}_{n-4}$. A few possible directions for future research are also given.