H. W. Gould and Jocelyn Quaintance
Products of numbers which obey a Fibonacci-type recurrence,
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Abstract

Let

$$Q_r(n) = G_{n+1}G_{n+2}\cdots G_{n+r} \tag{0.1}$$

$$\hat{Q}_r(n) = J_{n+1}J_{n+2}\cdots J_{n+r} \tag{0.2}$$

where, for various non-zero constants a, b, and c, one defines

$$G_m = aG_{m-1} + bG_{m-2} \tag{0.3}$$

$$J_m = aJ_{m-1} + bJ_{m-2} + cJ_{m-3}. \tag{0.4}$$

Through repeated iterations, one can show that

$$Q_r(n) = \sum_{j=1}^{r} R_j^r(a,b)G_{n+1}^{r+1-j}G_n^{j-1} \tag{0.5}$$

$$\hat{Q}_r(n) = \sum_{p=1}^{r-1}\sum_{q=1}^{r-p} R_{p,q}^r(a,b,c)J_{n+2}^p J_{n+1}^q J_n^{r-p-q}, \tag{0.6}$$

where the $R_j^r(a,b)$ and $R_{p,q}^r(a,b,c)$ are polynomials that obey a recurrence relation. This recurrence relation is a sum whose terms are binomial coefficients times monomials $a^l b^k$ for (0.5) or binomial coefficients times monomials $a^l b^k c^s$ for (0.6).