Jiří Klaška and Ladislav Skula
Periods of the Tribonacci Sequence Modulo a Prime $p \equiv 1 \pmod 3$,
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Abstract

Let the Tribonacci polynomial $t(x) = x^3 - x^2 - x - 1$ be irreducible over the Galois field $\mathbb{F}_p$ where p is an arbitrary prime such that $p \equiv 1 \pmod 3$ and let τ be any root of $t(x)$ in the splitting field K of $t(x)$ over $\mathbb{F}_p$. We prove that $\tau^{(p^2+p+1)/3} = 1$. Using this identity we show that the period $h(p)$ of the sequence $(T_n \bmod p)_{n=0}^{\infty}$ where T_n is the nth Tribonacci number divides $(p^2 + p + 1)/3$. Similar results will also be obtained for $t(x)$ being reducible over $\mathbb{F}_p$. In this case we prove that the period $h(p)$ divides $(q - 1)/3$ where q is the number of elements of the splitting field of $t(x)$ over $\mathbb{F}_p$ if and only if 2 is a cubic residue of $\mathbb{F}_p$.