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The Greatest Prime Factor and Recurrent Sequences,
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Abstract

The recurrent sequences considered in the present paper are prime sequences of the form $p_j = \mathrm{gpf}(a_1 p_{j-1} + a_2 p_{j-2} + \cdots + a_d p_{j-d} + a_0)$, where for any integer $x \geq 2$, we denote by $\mathrm{gpf}(x)$ the greatest prime factor of x. In the simple case of the 'GPF-Fibonacci' sequences corresponding to $d = 2$, $a_0 = 0$, and $a_1 = a_2 = 1$, we find that regardless of the initial conditions p_0 and p_1, all such sequences ultimately enter the cycle 7, 3, 5, 2. A computational exploration of the 'GPF-Tribonacci' analogue $d = 3$, $a_0 = 0$, and $a_1 = a_2 = a_3 = 1$ reveals four cycles of lengths, listed in the decreasing order of frequencies, 100, 212, 28 and 6, with the two larger cycles collecting more than 98% of the sequences as defined by the initial conditions p_0, p_1, and p_2. The paper concludes with a general ultimate periodicity conjecture and discusses its plausibility.