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A Lucas Type Theorem Modulo Prime Powers,
Fibonacci Quart. **51** (2013), no. 2, 142–146

Abstract

In this note we prove that

$$\binom{np^s}{mp^s+r} \equiv (-1)^{r-1}r^{-1}(m+1)\binom{n}{m+1}p^s \pmod{p^{s+1}}$$

where p is any prime, n, m, s and r are nonnegative integers such that $n \geq m$, $s \geq 1$, $1 \leq r \leq p^s - 1$ and r is not divisible by p. We derive a proof by induction using a multiple application of Lucas' Theorem and two basic binomial coefficient identities. As an application, we prove that a similar congruence for a prime $p \geq 5$ established in 1992 by D. F. Bailey holds for all primes p.