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Abstract

Let the Fibonacci words be $w_1 = 0$, $w_2 = 1$, $w_{n+1} = w_n w_{n-1}$ considered as integers expressed in binary. It is known that for $n \geq 2$ the numbers $0.\overline{w}_n = \frac{w_n}{2^{F_n} - 1}$ have the continued fraction $[0; 2^0, 2^1, 2^1, 2^2, 2^3, 2^5, \ldots, 2^{F_{n-2}}]$. We provide a simple proof using Fibonacci-type recurrences of compositions of linear functions. We apply this to several related recurrences.