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Sums of Reciprocals of Weighted Products of the Sine and and Cosine Functions,

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Abstract

In this paper, we define 12 families of finite sums that involve the sine/cosine functions. Four of these families are parametrized by j, and the remaining eight families are parametrized by j and k. In each of the aforementioned 12 families, the denominator of the summand contains a product of sine or cosine functions, and the length of this product is governed by the parameter j. As such, the length of the product in question can be made as large as we please.

In each of the 12 families of finite sums that we consider, there is a so-called *weight term* in the summand. For instance, in S_4 (defined in Section 2), the weight term is $(\frac{1}{2\cos j})^i$.