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#### Abstract

Let $F_{n}$ and $L_{n}$ be the $n$th Fibonacci and Lucas numbers, respectively. Let $\varphi(n)$ be the Euler totient function of $n$ and $\sigma_{k}(n)$ the sum of $k$ th powers of the positive divisors of $n$. Luca obtained the inequalities $\varphi\left(F_{n}\right) \geq F_{\varphi(n)}, \sigma_{0}\left(F_{n}\right) \geq F_{\sigma_{0}(n)}$, and $\sigma_{k}\left(F_{n}\right) \leq F_{\sigma_{k}(n)}$ for all $n, k \geq 1$. In this article, we extend Luca's result by replacing the function $\varphi$ by $\varphi_{k}$ and $J_{k}$, which are generalizations of $\varphi$. We also consider the corresponding results for $\varphi_{k}\left(L_{n}\right), L_{\varphi_{k}(n)}, J_{k}\left(L_{n}\right), L_{J_{k}(n)}, \sigma_{k}\left(L_{n}\right)$, and $L_{\sigma_{k}(n)}$.


