Jonathan J. Azose and Arthur T. Benjamin A Tiling Interpretation of the q-Binomial Coefficients, Fibonacci Quart. 58 (2020), no. 2, 99–125.

Abstract

We provide a combinatorial interpretation of the q-binomial and qmultinomial coefficients as counting weighted collections of tiled boards. Using this interpretation, we prove a new q-analogue to Lucas' Theorem and new q-analogues to identities on the sums of integer squares and cubes. Further proofs of known q-identities illustrate the use of proof elements including generating functions, recurrence relations, and signreversing involutions, all in the q-binomial context.