Richard J. McIntosh Congruences Involving Euler Numbers and Power Sums, Fibonacci Quart. 58 (2020), no. 4, 328–333.

Abstract

In her 1938 paper on congruences involving Bernoulli numbers and the quotients of Fermat and Wilson, Emma Lehmer expresses the residues modulo prime powers of many power sums in terms of Bernoulli numbers and sometimes Euler numbers. The Euler numbers often appear in the residues of alternating power sums. In this paper, we give a new congruence for determining the residues of Euler numbers modulo a prime p. This congruence involves about p/6 summands of an alternating power sum. Evaluating sums of reciprocal squares modulo p, we found that there are eight Euler irregular pairs (p, p - 3) with $p < 5 \times 10^9$.