King Shun Leung
φ-Expansions of Rationals,
Fibonacci Quart. 61 (2023), no. 2, 162–166.

Abstract

Let ϕ denote the golden ratio $(\sqrt{5}+1)/2$ and x be a positive rational number. We study how the ϕ -expansion of x can be found using some known results on Fibonacci numbers. We characterize those numbers in (0, 1) with finite ϕ -expansions. If $x \in \mathbb{Q} \cap (0, 1)$, we give a precise expression for its ϕ -expansion. In this case, the computation involves only simple operations on integers.