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Binomial Sums Involving Second-Order Linearly Recurrent Sequences, Fibonacci Quart. **62** (2024), no. 1, 57–64.

Abstract

Consider the sequences $(U_n : n \in \mathbb{N}_0)$ and $(V_n : n \in \mathbb{N})$ satisfying the second-order linear recurrences $U_n = pU_{n-1} + U_{n-2}$ and $V_n = pV_{n-1} + V_{n-2}$ with the initial conditions $U_0 = 0$, $U_1 = 1$, $V_0 = 2$, and $V_1 = p$. We explore the problem of evaluating binomial sums involving products consisting of entries in the U- and V-sequences. We apply a hypergeometric approach, inspired by Dilcher's work on hypergeometric identities for Fibonacci numbers, to obtain many new identities for sums involving U and V and products of binomial coefficients, including a non-hypergeometric analogue of Dixon's binomial identity.