DETERMINANTS RELATED TO 1979

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$$\begin{vmatrix} 1 & 9 \\ 7 & 9 \end{vmatrix} = -2 \cdot 3^3 \quad \text{and} \quad \begin{vmatrix} 1 & 9 \\ 9 & 7 \end{vmatrix} = -2 \cdot 37.$$

$$\begin{vmatrix} 1 & 9 & 7 & 9 \\ 9 & 1 & 9 & 7 \\ 7 & 9 & 1 & 9 \\ 9 & 7 & 9 & 1 \end{vmatrix} = -2^4 \cdot 3^2 \cdot 65 = -\begin{vmatrix} 1 & 9 & 7 & 9 \\ 9 & 7 & 9 & 1 \\ 7 & 9 & 1 & 9 \\ 9 & 1 & 9 & 7 \end{vmatrix}.$$

$$\begin{vmatrix} 1 & 9 & 7 & 9 \\ 9 & 7 & 9 & 0 \\ 7 & 9 & 0 & 0 \\ 9 & 0 & 0 & 0 \end{vmatrix} = 3^8 \cdot \begin{vmatrix} 1 & 9 & 7 & 9 \\ 0 & 1 & 9 & 7 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 1 \end{vmatrix}.$$

$$\begin{vmatrix} 1 & 9 & 7 & 9 \\ 9 & 7 & 9 & 7 \\ 7 & 9 & 7 & 9 \\ 9 & 7 & 9 & 1 \end{vmatrix} = -6^2(9^2 - 7^2) = -2^7 \cdot 3^2.$$

$$D = \begin{vmatrix} 1 & 9 & 7 & 9 \\ 9 & a & a & 7 \\ 7 & a & a & 9 \\ 9 & 7 & 9 & 1 \end{vmatrix} = (9^2 - 7^2) - 80a = 2^4(2^6 - 5a),$$

so for pertinent values of a, we have

a	0	1	7	9
D	2^{10}	$2^4 \cdot 59$	$2^4 \cdot 29 = 464$	$2^4 \cdot 19$

$$\begin{vmatrix} 9 & 1 & 9 \\ 7 & x & 7 \\ 9 & 1 & 9 \end{vmatrix} = 0, \quad \text{and } d = \begin{vmatrix} 1 & 9 & 7 \\ 9 & b & 9 \\ 7 & 9 & 1 \end{vmatrix} = 12(81 - 4b),$$

so for pertinent values of b, we have

b	0	1	7	9
d	$2^3 \cdot 3^5$	$2^2 \cdot 3 \cdot 79$	$2^2 \cdot 3 \cdot 59$	$2^2 \cdot 3^3 \cdot 5$

$$-\begin{vmatrix} 1 & 2 & 3 \\ 8 & 5 & 4 \\ 6 & 7 & 9 \end{vmatrix} = 1, \quad \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 9 & 8 \end{vmatrix} = 9, \quad \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 8 \\ 6 & 7 & 9 \end{vmatrix} = 7, \quad \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 8 & 9 & 7 \end{vmatrix} = 9.$$

The first nine-digit determinant becomes the third upon change of sign and reversal of the second row; the third becomes the second upon a one-step cyclic permutation of the digits 6, 7, 9, 8; and the second becomes the fourth upon reversal of the third row.

$$\begin{vmatrix} 1 & 4 & 8 \\ 9 & 2 & 7 \\ 5 & 6 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 4 & 8 \\ 7 & 2 & 6 \\ 5 & 9 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 4 & 7 \\ 8 & 2 & 6 \\ 5 & 9 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 4 & 7 \\ 9 & 2 & 6 \\ 5 & 8 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 4 & 7 \\ 8 & 2 & 5 \\ 6 & 9 & 3 \end{vmatrix}\begin{matrix} & \\ 8 & \\ 6 & 9 \end{matrix}$$

$$= 348 + 412 + 410 + 404 + 405 = 1979.$$

The first nine-digit determinant becomes the second upon a one-step counter-clockwise rotation of the 6, 7, 9 configuration; the second becomes the third upon interchange of 7 and 8; the third becomes the fourth upon interchange of 8 and 9; the fourth becomes the fifth upon a two-step rotation of the 9, 6, 8, 5 configuration. In the last determinant, the nine digits are in order of magnitude along a main diagonal and the two broken diagonals parallel to it.

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REITERATIVE ROUTINES APPLIED TO 1979

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(A) Sum the digits of the integer.

1 + 9 + 7 + 9 = 26, 2 + 6 = 8, the digital root of 1979.

(B) Compute the absolute difference of the integer and its reverse.

1979	7812	5625	360	297	495	99
9791	2187	5265	063	792	594	99
7812	5625	360	297	495	99	0

Seven operations to reach the inevitable 0.

(C) Add the integer and its reverse.

1979	11770	19481	37972	65945	120901
9791	07711	18491	27973	54956	109021
11770	19481	37972	65945	120901	229922

Six operations to reach a palindrome. Continuing the procedure for 18 more operations produces the palindrome 8813200023188.

(D) The Kaprekar routine. Arrange the digits in descending order, and from it subtract its reverse.

9971	8721	7443	9963	6642	7641
1799	1278	3447	3699	2466	1467
8172	7443	3996	6264	4176	6174

Six operations to reach Kaprekar's constant, the self-replicating 6174.

(E) The Collatz algorithm. If it is odd, triple it and add 1; if it is even, divide it by 2.

1979	530	143	233	1132	911	122
5938	265	430	700	566	2734	61
2969	796	215	350	283	1367	184
8908	398	646	175	850	4102	92
4454	199	323	526	425	2051	46
2227	598	970	263	1276	6154	23
6682	299	485	790	638	3077	70
3341	898	1456	395	319	9232	35
10024	449	728	1186	958	4616	106
5012	1348	364	593	479	2308	53
2506	674	182	1780	1438	1154	160
1253	337	91	890	719	577	80
3760	1012	274	445	2158	1732	40
1880	506	137	1336	1079	866	20
940	253	412	668	3238	433	10
470	760	206	334	1619	1300	5
235	380	103	167	4858	650	16
706	190	310	502	2429	325	8
353	95	155	251	7288	976	4
1060	286	466	754	3644	488	2
			377	1822	244	1

It takes 143 operations to reach the inevitable 1.