A NOTE ON A PAPER OF G. H. WEISS AND M. DISHON

Helmut Prodinger

The John Knopfmacher Centre for Applicable Analysis and Number Theory
School of Mathematics, University of the Witwatersrand, P.O. Wits 2050 Johannesburg, South Africa
e-mail: helmut@gauss.cam.wits.ac.za

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In [2], Weiss and Dishon improved an earlier result of Narayana and Kreweras, by proving that for $r, s \geq 1$

$$[u^r v^s]\frac{1-u-v-\sqrt{1-2(u+v)+u-v)^2}}{2} = \frac{1}{r+s-1}\binom{r+s-1}{r}\binom{r+s-1}{s}$$

(the notation $[u^r v^s]f(u,v)$ refers to the coefficient of $u^r v^s$ in the power series expansion of $f(u,v)$).

However, the *really easy* method in this context is the *Lagrange inversion formula* as will be demonstrated now. If $S = (1-u-v-\sqrt{1-2(u+v)+(u-v)^2})/2$, then $S^2+(u+v-1)S+uv = 0$, or

$$u = \frac{S}{\Phi(S)} \text{ with } \Phi(S) = \frac{v+S}{1-v-S}.$$

Now the Lagrange inversion formula tells us (see, e.g., [1]) that

$$[u^r]S = \frac{1}{r}\left[S^{r-1}\right](\Phi(S))^r,$$

or, with $v = St$,

$$\begin{aligned}
[u^r v^s]S &= \frac{1}{r}[S^{r-1}v^s]\left(\frac{v+S}{1-v-S}\right)^r \\
&= \frac{1}{r}[S^{r+s-1}t^s]\left(\frac{S(1+t)}{1-S(1+t)}\right)^r \\
&= \frac{1}{r}[t^s](1+t)^r[S^{s-1}](1-S(1+t))^{-r} \\
&= \frac{1}{r}[t^s](1+t)^{r+s-1}\binom{r+s-2}{s-1} \\
&= \frac{1}{r}\binom{r+s-1}{s}\binom{r+s-2}{s-1},
\end{aligned}$$

which is clearly equivalent to the statement to be proved.

REFERENCES

[1] R. Stanley. *Enumerative Combinatorics.* Volume 2. Cambridge University Press, Cambridge, 1999.

[2] G.H. Weiss and M. Dishon. "A Method for the Evaluation of Certain Sums Involving Binomial Coefficients." *The Fibonacci Quarterly* **14** (1976): 75-77.

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