

Thomas Koshy's Catalan Numbers with Applications

Reviewed by

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Tom Koshy's *Catalan Numbers with Applications* should prove to be of great value to professional mathematicians and to other professionals who encounter these numbers in their respective fields of activity, be that in science or in engineering. The book is aimed at a large student readership as well because it contains a lot of worked out examples that increase the impact of the main theorems and clarify many details of the given proofs. The book should thus be an important reading reference for courses in number theory. Furthermore, the book should prove of interest to non-professional number theory enthusiasts because it is eminently clear and self-contained.

Koshy begins his presentation by reviewing some of the basic properties of the binomial coefficients (chapter 1). Then in the next three chapters, the author introduces the central binomial number and he describes a variety of situations where that number occurs, stressing its general significance and pervasiveness. In chapter 5, after having set the stage in his usual gradual manner, Koshy defines Catalan numbers and spends the remaining twelve chapters relating these numbers to the different contexts where they pop up. We learn, in chapters 8 to 12, that Catalan numbers show up in number theoretical studies of graphs (trees, forests, etc.), of lattice paths, of partitions, of algebra, of sports, and of Pascal's triangle. More specialized topics that pertain to the Catalan numbers per se are then considered, including their divisibility, parity and primality in chapter 13. A Catalan triangle is introduced and interpreted geometrically in chapter 14 and links to binary word families and to tribonomial coefficients are discussed in chapters 15 to 17. The book ends by a short chapter on a generalization of Catalan numbers. There is also an appendix on the principle of mathematical induction and the basics of combinatorics, recursions, generating functions and congruences, all elements and definitions that are required to follow the presentation in the text. A second appendix gives a list of the first one hundred Catalan numbers. Finally the bibliography collected by Koshy is extensive and contains more than 300 references to other texts or to the research literature.

This book is well written, well documented and user-friendly, qualities that are a trademark of Koshy's writing style, as can be seen from *Fibonacci and Lucas Numbers* (2001) and *Elementary Number Theory* (2001), among others. Koshy's style is lively, clear and precise. The underlying concepts, such as graphs and trees are presented with great clarity and ease. Koshy's tendency to present half a page or more of biographical notes about the major contributors to the field represent informative and delightful asides from the main text and authors of special results are given the courtesy of having their workplace affiliation noted.

THE FIBONACCI QUARTERLY

Koshy's book on Catalan numbers contains an impressive number of proofs of theorems, of results, and of references and it should thus provide a valuable source of information for students, professionals and amateurs alike. I recommend the book to all those with an interest in numbers!

MSC2000: 11B39, 33C05