1.1. Every third Fibonacci number is even; every fourth, a multiple of 3; every fifth, a multiple of 5.

1.2. Every nth Fibonacci number is divisible by F_n.

1.3. d is the greatest common divisor of a and b if and only if

(i) $d \mid a$, $d \mid b$, and

(ii) if $k \mid a$ and $k \mid b$, then $k \mid d$.

1.4.
$$\begin{aligned} 987 &= 9\times 100 + 8\times 10 + 7 \\ &= 9\times(99+1) + 8\times(9+1) + 7 \\ &= (9\times 99) + \underline{9} + (8\times 9) + \underline{8} + \underline{7} \\ &= 9\times(99+8) + (9+8+7) \end{aligned}$$

Since the first term is a multiple of 9, 987 will be divisible by 9 if and only if $9 + 8 + 7$ is a multiple of 9.

1.5. One need only try the primes not exceeding the greatest integer equal to or less than the square root of 4181 or 64 +. How many primes are less than 64?

1.6. The number in the i-th row and j-th column is given by the formula

$$(2j+1)\,i + j.$$

(a) If N is in **S**, then $2N + 1$ is of the form

$$2\{(2j+1)i + j\} + 1 = 4ij + 2i + 2j + 1 = (2i+1)(2j+1)$$
$$= \text{a composite number.}$$

(b) To prove that if N is not in **S**, then $2N + 1$ is a prime we consider an equivalent statement (called the contrapositive): If $2N + 1$ is not a prime, then N is in **S**.

Now if $2N + 1$ is not a prime, it has an odd factor $2i' + 1$ (which is between 1 and $2N + 1$). Thus

$$2N + 1 = (2j'+1)(2i'+1) = 2\{(2j'+1)i' + j'\} + 1$$

or $N = (2j'+1)i' + j'$, i.e., N lies in row i' and column j'.

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