$$F_{n-1}F_{n+1} - F_n^2 = (-1)^n \qquad (n \geq 2)$$

implies

$$F_{n-1}^2 \equiv (-1)^n \pmod{F_n} .$$

If n is even $(n \geq 4)$, we have $F_{n-1}^2 \equiv 1 \pmod{F_n}$ and $u_n = 2n$. If n is odd $(n > 4)$, $F_{n-1}^2 \equiv -1 \pmod{F_n}$ and $u_n = 4n$.

From the above, it is obvious that $N = 1$ is the smallest positive integer for which (iii) holds for all $n = 1, 2, \cdots$. It is interesting to note that

$$\{u_n | n = 1, 2, \cdots\} \cap \{F_n | n = 1, 2, \cdots\} = \{F_1, F_4, F_6, F_9, F_{12}, \cdots\} .$$

[Continued from page 282.]

NOTE ON SOME SUMMATION FORMULAS

by

$$\frac{\displaystyle\prod_{i=1}^{s_0+s_1+s_2+\cdots} (k + s_1 + 2s_2 + 3s_3 + \cdots + i)}{s_0!\,s_1!\,s_2!\,\cdots} .$$

REFERENCE

1. L. Carlitz, "Some Summation Formulas," Fibonacci Quarterly, Vol. 9 (1971), pp. 28-34.