FIBONACCI NUMBERS AND WATER POLLUTION CONTROL

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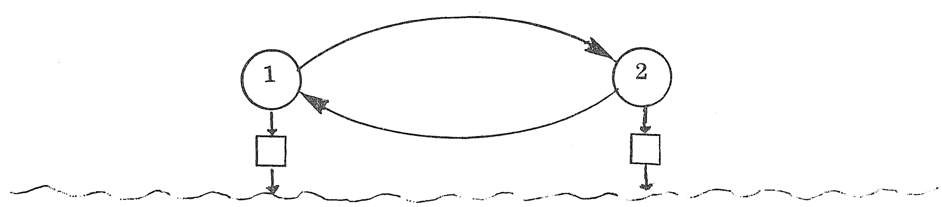
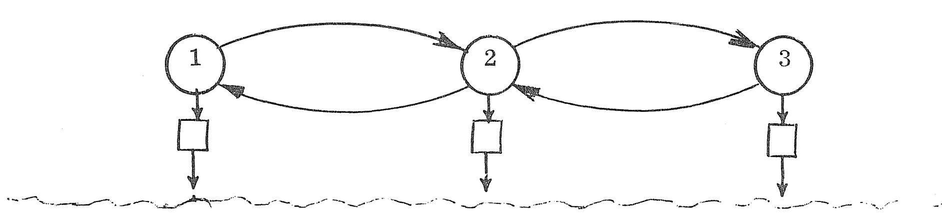
Consider a number of cities along a major water course which discharge presently their wastes untreated to the stream. To control the polution of the waters they have the obligation to build treatment plants. The major question is, where should one build these plants to minimize the cost of pollution control? Construction as well as operation costs of treatment plants exhibit large economies of scale, and therefore it is generally economically advantageous to build one or more central treatment plants. Given one possible location for a treatment plant for each city, and the possibility to transport the waste waters from any city to another one, the problem arises of how many possible solutions there are. Due to the economies of scale it is known that it would not be economical to "split" the waste flow of one city, that is, transport part of the waste upstream and part of it downstream.

Consider two cities only:



The number of possible solutions is $A(2) = 3$; namely, a treatment plant at each city, one treatment plant at city 1, and finally, one treatment plant at city 2.

Consider now 3 cities:



The interconnecting sewers between the cities are the only decision variables. Let a zero indicate no transport between cities, a 1 for upstream transport of wastes, and a 2 for downstream transport of wastes. For n cities there are $(n - 1)$ connecting sewers between the cities, each of which may take on 3 values. So the total number of solutions would be 3^{n-1} were it not for the economic requirement that a city may not simultaneously transport wastes upstream and downstream. For three cities, the total number of solutions may be represented as follows:

00 01 02 10 11 12 20 21* 22

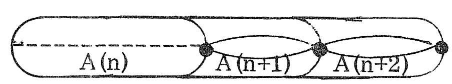
The 8^{th} solution indicated by an asterisk is ruled out, since we do not allow transport of waste water from city 2 simultaneously to 1 and 3. And thus the total number of economical solutions will be $A(3) = 8$.

Consider now n cities:

Let $A(n + 2)$ stand for the number of solutions for $n + 2$ cities, $A(n + 1)$ for the number of solutions to $n + 1$ cities, and $A(n)$ for the solutions for n cities.

Then, the following recursive relation can be established:

$$A(n + 2) = 3A(n + 1) - A(n) .$$



This relation may be deduced by the following reasoning. Given the value of $A(n + 1)$, the adding of one city increases the number of solutions to

$$3 \cdot A(n + 1)$$

since the new added sewer may assume the values of 0, 1, or 2. However, of this total number there are some which are not economical, namely, all those which end in a 2 1 sequence. But the number of those is exactly $A(n)$.
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