A NUMBER PROBLEM

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In a recent note (this Journal, Vol. 4 (1971), p. 195), Wlodarski gives two solutions for the problem of determining the smallest number ending in 6 such that the number formed by moving the 6 to the front of the number is equivalent to multiplying the given number by 6. Here we give a more compact solution and answer.

If the given number is represented by

$$N = a_0 \cdot 10^n + a_1 \cdot 10^{n-1} + \cdots + a_{n-1} \cdot 10 + 6 ,$$

then

$$I = 6[10^{n+1} - 1]/59 .$$

By Fermat's theorem, $a^{p-1} \equiv 1 \pmod{p}$, and thus

(1) $$I = 6[10^{58} - 1]/59 .$$

Since it can be shown that $10^{29} \equiv -1 \pmod{59}$, it follows that the number in (1) is the least one with the desired property.

There is no need to assume the number ends in 6. For if the number ended in 7, then

$$I = 7(10^{58} - 1)/59$$

would satisfy the deleted conditions but would be larger.

A similar example and solution for the case 6 is replaced by 9 had been given by the author previously.[1]

[1]M. S. Klamkin, "On the Teaching of Mathematics so as to be Useful," Educ. Studies in Math., Vol. 1 (1968), p. 140.