no recurrence (except the identity recurrence (F_n)) has a norm dividing p. We shall proceed by induction.

For $k = 1$, the theorem is obviously true. Assume truth for all exponents not greater than k. Then there are two recurrences of norm p^k which factor uniquely, and since $(A_n)^k$ and $(A_n^*)^k$ are factorizations of the recurrences of norm p^k, they are unique factorizations. Multiplying $(A_n)^k$ and $(A_n^*)^k$ by each of the recurrences of norm p and using (7), we get the products

$$(A_n)^{k+1}, \qquad (A_n^*)^{k+1}, \qquad (A_n)^k(A_n^*) = N(A)(A_n)^{k-1}, \qquad \text{and} \qquad (A_n^*)^k(A_n) = N(A)(A_n^*)^{k-1},$$

and the last two products fail to satisfy the requirement that the terms have no common factor. Thus, $(A_n)^{k+1}$ and $(A_n^*)^{k+1}$ are two factorizations of recurrences of norm p^{k+1}, and they are the only two meeting the requirement that the terms of the product have no common factor. Since there are two recurrences of norm p^{k+1} (see [2]), $(A_n)^{k+1}$ and $(A_n^*)^{k+1}$ must be their factorizations. This completes the proof.

REFERENCES

1. Brother U. Alfred, "On the Ordering of Fibonacci Sequences," *The Fibonacci Quarterly,* Vol. 1, No. 4 (December, 1963), pp. 43–46.
2. T.W. Cusick, "On a Certain Integer Associated with a Generalized Fibonacci Sequence," *The Fibonacci Quarterly,* Vol. 6, No. 2 (April, 1968), pp. 117–126.
3. P. Naor, "Letter to the Editor," *The Fibonacci Quarterly,* Vol. 3, No. 4 (December, 1965), pp. 71–73.
4. Dmitri Thoro, "An Application of Unimodular Transformations," *The Fibonacci Quarterly,* Vol. 2, No. 4 (December, 1964), pp. 291–295.
5. Oswald Wyler, "On Second-Order Recurrences," *American Math. Monthly,* 72 (1965) pp. 500–506.

****

A NOTE ON FERMAT'S LAST THEOREM

DAVID ZEITLIN
Minneapolis, Minnesota

In this note, $n, m, x, y,$ and z are all positive integers, with $x < y < z$.

Theorem 1. For $n \geqslant 2$, the equation $x^n + y^n = z^n$ has no solutions whenever $x + ny \leqslant nz$.

Corollary. For $m \geqslant 1$ and $n \geqslant 2$, $x^{mn} + y^{mn} = z^{mn}$ has no solutions whenever $x^m + ny^m \leqslant nz^m$.

Proof. Suppose $x^n + y^n = z^n$ has a solution with $y = x + a$, $z = x + b$, where $b > a > 0$ are integers. Then, by using the binomial theorem, we have

$$x^n = z^n - y^n = (x+b)^n - (x+a)^n = \sum_{i=0}^{n} \binom{n}{i} x^{n-i}(b^i - a^i) = nx^{n-1}(b-a) + Q(n,x,b,a), \quad Q > 0.$$

Thus

$$x^{n-1}(x - n(b-a)) = Q,$$

and so $x - n(b-a) > 0$ is a necessary condition for a solution. Since

$$b - a = (x+b) - (x+a) = z - y, \quad x - n(z-y) \leqslant 0$$

is the stated result.

REMARKS. Since $nz < ny + x$ is a necessary condition for a solution and since $y < z$, we see that

[Continued on Page 402.]