To form the fractions in the intervals $(1,2), (2,3), (3,5), \cdots$, write the reciprocals in reverse order of the fractions in $(1/2, 1)$ in $f \cdot f_{n+1}$, of $(1/3, 1/2)$ in $f \cdot f_{n+2}, \cdots$, respectively. This gives $f \cdot f_n$ as far as we want it.

In fact, one of the purposes of investigating the symmetries of Farey Fibonacci sequences was to develop easy methods to form them.

REFERENCE

1. Krishnaswami Alladi, "A Farey Sequence of Fibonacci Numbers," *The Fibonacci Quarterly,* Vol. 13, No. 1 (Feb. 1975), pp.

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A SIMPLE PROOF THAT PHI IS IRRATIONAL

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Most proofs of the irrationality of phi, the golden ratio, involve the concepts of number fields and the irrationality of $\sqrt{5}$. This proof involves only very simple algebraic concepts.

Denoting the golden ratio as ϕ, we have

$$\phi^2 - \phi - 1 = 0 .$$

Assume $\phi = p/q$, where p and q are integers with no common factors except 1. For if p and q had a common factor, we could divide it out to get a new set of numbers, p' and q'.

Then

$$(p/q)^2 - p/q - 1 = 0$$

$$(p/q)^2 - p/q = 1$$

$$p^2 - pq = q^2$$

$$p(p-q) = q^2 \tag{1}$$

Equation (1) implies that p divides q^2, and therefore, p and q have a common factor. But we already know that p and q have no common factor other than 1, and p cannot equal 1 because this would imply $q = 1/\phi$, which is not an integer. Therefore, our original assumption that $\phi = p/q$ is false and ϕ is irrational.

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