

[Continued from Page 173.]

1-1-29-29	2-2-15-15	3-3-10-10	4-4-8-8
5-5-6-6	1-2-29-15	1-15-29-2	1-3-29-10
1-10-29-3	1-4-29-8	1-8-29-4	1-5-29-6
1-6-29-5	2-3-15-10	2-10-15-3	2-4-15-8
2-8-15-4	2-5-15-6	2-6-15-5	3-4-10-8
3-8-10-4	3-5-10-6	3-6-10-5	4-5-8-6
4-6-8-5			

For any given values of p and q , it is not difficult to determine all such non-periodic sequence types.

A MODIFIED TYPE OF SEQUENCE

The students created another type of sequence in which the multipliers interchange their position from one step to the next. Thus for $T_7(2,1)$ where the multipliers are 2 and 1 and then 1 and 2, starting with 2,5,7, the next term is $[(2 * 7 + 5)/2] * 10$; the following term is $[(1 * 10 + 2 * 7)/5] * 5$, etc. The periods for $T_7(2,1)$ were found to be:

A: 3,3,2,3,3,5,4,5	E: 2,4,2,4	I: 4,2,4,3,6,4,6,3
B: 3,3,3,3,...	F: 4,3,4,3	J: 5,1,6,3,15,6,12,2
C: 3,2,3,3,5,5,5,3	G: 2,5,2,5	K: 5,3,5,3
D: 2,3,2,4,3,5,3,4	H: 3,1,3,3,9,7,9,3	L: 6,1,5,2,12,6,15,3
		M: 3,4,3,4

It should be noted that a period 3,4,3,4 in this setup is not the same as a period 4,3,4,3. Evidently this opens up another broad area for investigation.

CONCLUSION

The purpose of reporting this research is in the first instance to offer a model of cooperative effort where teacher and students work on a problem of unknown potential. Secondly, we feel that we have just scratched the surface and wish to open up the many possibilities to interested parties, especially people who have access to computer time.

Just a few of the points for investigation may be indicated.

- (1) A major conjecture to be proved: For sequences of type (p,q) , if $p \geq q$, all sequences are periodic; for sequences with $p < q$, some sequences are periodic and some non-periodic of the type mentioned in this summary.
- (2) Additional work on possible and non-possible period lengths.
- (3) Determining the lengths of periods for given values of p and q .
- (4) In the case of periodic sequences, finding upper bounds for the values of terms in the periods.
- (5) Arriving at additional generalized sequences for other values of p and q than (p,p) , $(p+1,p)$, $(p,p+1)$.
- (6) Modifying the work to include more terms in the numerator with a corresponding number of multipliers.
- (7) Studying the least integer functions which involve non-linear combinations of the previous terms.

NOTE

The complete report of which this article is a summary consists of 54 pages. It may be obtained for \$2.50 by writing the Managing Editor, Brother Alfred Brousseau, St. Mary's College, Moraga, California 94575.

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