of the numbers $0, 1, 4, 15, 56, 209, \cdots$. If $M = 110 = 2\cdot5\cdot11$ then $m = 2\cdot\frac{6}{2}\cdot\frac{10}{2} = 30$ so $110 \mid Y_{30}$. If $M = 18 = 2\cdot3^2$ then $m = 2\cdot3^2 = 18$ so $18 \mid Y_{18}$.

EXAMPLE 2. $X^2 - 2Y^2 = 1$ then $X_1 = 3,\ Y_1 = 2,\ \Delta = 32.$

The sequence $Y_0, Y_1, Y_2, \cdots$ consists of the numbers $0, 2, 12, 70, \cdots$ (which are Pell numbers with even subscript). The rank of apparition of any number M is less than M.

REMARK

If $b \neq \pm 1$ the theorem will generally not be valid; e.g., on taking $a = 4, b = 6, R_1 = 1$ any number M containing the factor 3 will not divide a member of the sequence.

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FIBONACCI AND LUCAS SUMS IN THE r-NOMIAL TRIANGLE

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ABSTRACT

Closed-form expressions not involving $c_n(p,r)$ are derived for

(1) $$\sum_{n=0}^{p(r-1)} c_n(p,r) f_{bn+j}^m(x)$$

(2) $$\sum_{n=0}^{p(r-1)} c_n(p.r) \ell_{bn+j}^m(x)$$

(3) $$\sum_{n=0}^{p(r-1)} c_n(p,r)(-1)^n f_{bn+j}^m(x)$$

(4) $$\sum_{n=0}^{p(r-1)} c_n(p,r)(-1)^n \ell_{bn+j}^m(x),$$

where $c_n(p,r)$ is the coefficient of y^n in the expansion of the r-nomial

$$(1 + y + y^2 + \cdots + y^{r-1})^p, \qquad r = 2, 3, 4, \cdots, \qquad p = 0, 1, 2, \cdots,$$

and $f_n(x)$ and $\ell_n(x)$ are the Fibonacci and Lucas polynomials defined by

$$f_1(x) = 1, \qquad f_2(x) = x, \qquad f_n(x) = xf_{n-1}(x) + f_{n-2}(x);$$
$$\ell_1(x) = x, \qquad \ell_2(x) = x^2 + 2, \qquad \ell_n(x) = x\ell_{n-1}(x) + \ell_{n-2}(x).$$

Fifty-four identities are derived which solve the problem for all cases except when both b amd m are odd; some special cases are given for that last possible case. Since $f_n(1) = F_n$ and $\ell_n(1) = L_n$, the n^{th} Fibonacci and Lucas numbers respectively, all of the identities derived here automatically hold for Fibonacci and Lucas numbers. Also, $f_n(2) = P_n$, the n^{th} Pell number. These results may also be extended to apply to Chebychev polynomials of the first and second kinds.

The entire text of this 51-page paper is available for \$2.50 by writing the Managing Editor, Brother Alfred Brousseau, St. Mary's College, Moraga, California 94575.

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