to both sides of the result which leads to

$$k_1^2F_n^2 + 2k_1k_2F_nF_{n-1} + k_2^2F_{n-1}^2 = k_1^2F_n^2 + k_2F_n^2 + k_1k_2F_nF_{n-1} + (-1)^{n+1}k_2^{n+1} \quad (20a)$$

It is easily seen that

$$F_{n+2} = k_1F_{n+1} + k_2F_n = k_1^2F_n + k_1k_2F_{n-1} + k_2F_n,$$

and combining this equation with (20a), we have

$$(k_1F_n + k_2F_{n-1})^2 = F_{n+1}^2 = F_{n+2}F_n + (-1)^{n+1}k_2^{n+1}. \quad (20b)$$

In the same way we found (20b), we proceed step-by-step (with added induction) and prove that the identities in (19) and (19a) are correct.

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PYTHAGOREAN TRIANGLES

DELANO P. WEGENER
Central Michigan University, Mount Pleasant, Michigan 48858
and
JOSEPH A. WEHLEN
Ohio University, Athens, Ohio 45701

ABSTRACT

The first section of "Pythagorean Triangles" is primarily a portion of the history of pythagorean triangles and related problems. However, some new results and some new proofs of old results are presented in this section. For example, Fermat's Theorem is used to prove:

Levy's Theorem. If (x,y,z) is a pythagorean triangle such that $(7,x) = (7,y) = 1$, then 7 divides $x + y$ or $x - y$.

The historical discussion makes it reasonable to define pseudo-Sierpinski triangles as primitive pythagorean triangles with the property that $x = z - 1$, where z is the hypotenuse and x is the even leg. Whether the set of pseudo-Sierpinski triangles is finite or infinite is an open question. Some elementary, but new, results are presented in the discussion of this question.

An instructor of a course in Number Theory could use the material in the second section to present a coherent study of Fermat's Last Theorem and Fermat's method of infinite descent. These two results are used to prove the following familiar results.

(1A) *No pythagorean triangle has an area which is a perfect square.*

(2A) *No pythagorean triangle has both legs simultaneously equal to perfect squares.*

(3A) *It is impossible that any combination of two or more sides of a pythagorean triangle be simultaneously perfect squares.*

If 2 is viewed as a natural number for which Fermat's Last Theorem is true, then the following are obvious generalizations of 1A, 2A, and 3A.

(1B) *If* k *is an integer for which Fermat's Last Theorem holds, then there is no primitive pythagorean triangle whose area is a* k^{th} *power of some integer.*

(2B) *If* k *is some integer for which Fermat's Last Theorem is true, then there is no pythagorean triangle with the legs both equal to* k^{th} *powers of natural numbers.*

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