Therefore mathematical induction yields the result. Q.E.D.

An immediate consequence of Corollary 1 and Lemma 2 is

Corollary 2. Either $D(n)$ is identically zero or never zero.

Zierler proves the following [2].

Lemma 3. Let $f(x)$ be a characteristic polynomial over the field F for the sequence

$$V = \{v_n\} \subseteq F, \qquad V \neq 0,$$

and let $g(x)$ be the minimum polynomial for V. Then

(i) $$g(x) \mid f(x),$$

(ii) $h(x)g(x)$ is also a characteristic polynomial for V, where $h(x)$ is any monic polynomial over F.

To complete the proof of Theorem 1 we note that Lemma 3 implies that V satisfies a lower order recursion if and only if some $f_k(x)$ as defined in (4) is a characteristic polynomial for V. But then Lemma 2 and Corollary 2 imply that V satisfies a lower order recursion if and only if $D(0) = 0$.

REFERENCES

1. M. Hall, "An Isomorphism Between Linear Recurring Sequences and Algebraic Rings," *Amer. Math. Monthly*, 44 (1938), pp. 196–217.
2. N. Zierler, "Linear Recurring Sequences," *J. Soc. Indust. Appl. Math.*, 7 (1959), pp. 31–48.

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A FIBONACCI PLEASANTRY

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In the Fibonacci sequence $F_0 = 0, F_1 = 1, \cdots, F_n = F_{n-1} + F_{n-2}$, list the sums $F_n + n$ in ascending order of n and note the second differences. Do the same with $F_n - n$.

0 + 0 =	0			0 − 0 =	0		
		> 2				> 0	
1 + 1 =	2		> −1	1 − 1 =	0		> −1
		> 1				> −1	
1 + 2 =	3		> 1	1 − 2 =	−1		> 1
		> 2				> 0	
2 + 3 =	5		> 0	2 − 3 =	−1		> 0
		> 2				> 0	
3 + 4 =	7		> 1	3 − 4 =	−1		> 1
		> 3				> 1	
5 + 5 =	10		> 1	5 − 5 =	0		> 1
		> 4				> 2	
8 + 6 =	14		> 2	8 − 6 =	2		> 2
		> 6				> 4	
13 + 7 =	20		> 3	13 − 7 =	6		> 3
		> 9				> 7	
21 + 8 =	29		> 5	21 − 8 =	13		> 5
		> 14				> 12	
34 + 9 =	43		> 8	34 − 9 =	25		> 8
		> 22				> 20	
55 + 10 =	65		> 13	55 − 10 =	45		> 13

[Continued on page 41.]