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LETTER TO THE EDITOR

October 13, 1975

Dear Professor Hoggatt:

It was with some surprise that I read Miss Ada Booth's article "Idiot's Roulette Revisited" in the April 1975 issue of *The Fibonacci Quarterly.* The problem she discusses—given N places circularly arranged and successively casting out the C^{th} place, determine which will be the last remaining place—is quite old and commonly referred to as the Josephus problem. The name alludes to a passage in the writings of Flavius Josephus [7], a Jewish historian who relates how after the fall of Jotapata, he and forty other Jews took refuge in a nearby cave, only to be discovered by the Romans. In order to avoid capture, everyone in the group, save Josephus, resolved on mass suicide. At Josephus' suggestion, lots were drawn, and as each man's lot came up, he was killed. By means not made clear in the passage, Josephus ensured that the lots of himself and one other were the last to come up, at which point he persuaded the other man that they should surrender to Vespasian.

Bachet [2], in one of the earliest works on recreational mathematics, proposed a definite mechanism by which this could have been accomplished: all forty-one people are placed in a circle, Josephus placing himself and the other man at the 16^{th} and 31^{st} places; every third person is then counted off and killed. This is, of course, a special case of the question Miss Booth considers.

Miss Booth's iterative solution to the general problem was apparently first discovered by Euler [5] in 1771 and then rediscovered by P. G. Tait [9], the English physicist and mathematician, in 1898. Tait points out that the method enables one to calculate the last r places to be left, not merely the last as in Miss Booth's article. Although Euler and Tait content themselves with demonstrating how the iterative solution works and do not actually derive the formula for Miss Booth's sequence of "subtraction numbers," in the 1890's Schubert and Busche [8, 4] derived a formula for this sequence (slightly modified) via a wholly different attack on the problem ("Oberreihen"). (Ahrens [1] has an excellent description of this work, as well as a comprehensive review of the history of the problem. Ball and Coxeter [3] briefly touch on the problem but omit any mention of the work of Schubert and Busche.)

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