A slight extension of the foregoing argument provides another proof of the main theorem of Wyler [10]. In fact, Wyler [10, Theorem 4] is valid for every purely periodic second-order Lucas sequence over a commutative ring with 1 satisfying the following two properties: 1 + 1 is not a zero divisor, and $u^2 = 1$ implies either $u = 1$ or $u = -1$.

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LETTER TO THE EDITOR

GENERALIZED FIBONACCI NUMBERS AND UNIFORM DISTRIBUTION MOD 1

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In the following I want to comment on a paper by William Webb concerning the distribution of the first digits of Fibonacci numbers [1] and to give a partial answer to some questions raised by the author. In fact, restriction to Fibonacci-related sequences makes it possible to obtain a number of results. (F_n) or 1, 1, 2, 3, 5, ⋯ stands for the sequence of Fibonacci numbers.

Theorem 1. Let k be an integer different from 0. Then the sequence $(\log F_n^{1/k})$ is uniformly distributed mod 1 (abbreviated u.d. mod 1).

Proof. We apply a classic result of J. G. van der Corput: Let (u_n) be a sequence of real numbers. If

$$\lim_{n\to\infty} (u_{n+1} - u_n)$$

exists and is irrational, then the sequence (u_n) is u.d. mod 1. See [2], p. 28.

Now set $u_n = \log F_n^{1/k}$. Then

$$u_{n+1} - u_n = \log F_{n+1}^{1/k} - \log F_n^{1/k} = \frac{1}{k} \log \frac{F_{n+1}}{F_n} ,$$

which tends to

[Continued on page 253.]