$$\begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix}\begin{pmatrix} \phi & 1 \\ 1 & -\phi \end{pmatrix} = \begin{pmatrix} \phi & 1 \\ 1 & -\phi \end{pmatrix}\begin{pmatrix} \phi^n & 0 \\ 0 & \phi'^n \end{pmatrix}.$$

Multiplying out gives

$$\begin{pmatrix} \phi F_{n+1} + F_n & F_{n+1} - \phi F_n \\ \phi F_n + F_{n-1} & F_n - \phi F_{n-1} \end{pmatrix} = \begin{pmatrix} \phi^{n+1} & \phi'^n \\ \phi^n & \phi'^{(n-1)} \end{pmatrix} .$$

Equating corresponding terms results in the following equivalent system of equations:

$$\phi F_{n+1} + F_n = \phi^{n+1}$$
$$F_{n+1} - \phi F_n = \phi'^n$$
$$\phi F_n + F_{n-1} = \phi^n$$
$$F_n - \phi F_{n-1} = \phi'^{(n-1)} .$$

Solving the second equation for F_{n+1} and substituting this into the first equation, gives

$$\phi(\phi'^n + \phi F_n) + F_n = \phi^{n+1}.$$

Multiplying through by $-\phi'$ gives

$$\phi'^n + \phi F_n - \phi' F_n = \phi^n .$$

Finally, solving for F_n gives the desired result:

$$F_n = \frac{\phi^n - \phi'^n}{\phi - \phi'} .$$

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From (9a) and (9b), we obtain

(10a) $$\sum_{n=-\infty}^{\infty} F_{(2k+1)n} J_n(x) = 0$$

and

(10b) $$\sum_{n=-\infty}^{\infty} F_{(2k+1)n-1} J_n(x) = \exp\left(\frac{x}{2} L_{2k+1}\right) .$$

Equations (10a) and (10b) can be combined in the following equation, as may be shown by induction

(11) $$\sum_{n=-\infty}^{\infty} F_{(2k+1)n+m} J_n(x) = F_m \exp\left(\frac{x}{2} L_{2k+1}\right) .$$

With $k = 0$ and $m = 1$, (11) becomes

$$\sum_{n=-\infty}^{\infty} F_{n+1} J_n(x) = \exp \frac{x}{2} .$$

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