$$U_{(p-1)/2g} \equiv 0 \pmod p \quad\text{and}\quad (-c)^{(p-1)/2g} \equiv 1 \pmod p,$$

then $k(H,p) = 2(p-1)/g$.

Proof. Let us use (13) to obtain

$$U_{(p-1)/g} \equiv 0 \pmod p \quad\text{and}\quad U_{\{(p-1)/g\}+1} \equiv -1 \pmod p.$$

Then it is easy to show that

(17) $$U_{2(p-1)/g} \equiv 0 \pmod p \quad\text{and}\quad U_{\{2(p-1)/g\}+1} \equiv \pmod p$$

when we get

(18) $$H_{2(p-1)/g} \equiv Q \pmod p \quad\text{and}\quad H_{\{2(p-1)/g\}+1} \equiv P \pmod p$$

and the desired result follows.

Analogously, we state the following theorems.

Theorem g. For primes of the form $2g(2t+1)-1$, where $t \equiv h \pmod{10}$ and $4gh+2g-1 \equiv \pm 3 \pmod{10}$, if

$$U_{\{(p+1)/2g\}+1} + cU_{\{(p+1)/2g\}-1} \equiv 0 \pmod p \quad\text{and}\quad c^{(p+1)/2g} \equiv 1 \pmod p,$$

then $k(H,p) = (p+1)/g$.

Theorem h. For primes of the form $4gt-1$, where $t \equiv h \pmod{10}$ and $4gh-1 \equiv \pm 3 \pmod{10}$, if

$$U_{(p+1)/2g} \equiv 0 \pmod p \quad\text{and}\quad (-c)^{(p+1)/2g} \equiv 1 \pmod p,$$

then $k(H,p) = (p+1)/g$.

Theorem i. For primes of the form $2g(2t+2)-1$, where $t \equiv h \pmod{10}$ and $4g+4gh-1 \equiv \pm 3 \pmod p$, if

$$U_{\{(p+1)/2g\}-1} + cU_{\{(p+1)/2g\}-1} \equiv 0 \pmod p \quad\text{and}\quad (-c)^{(p+1)/2g} \equiv 1 \pmod p,$$

then $k(H,p) = 2(p+1)/g$.

Theorem j. For primes of the form $2g(2t+1)-1$, where $t \equiv h \pmod{10}$ and $4gh+2g-1 \equiv \pm 3 \pmod{10}$, if

$$H_{(p+1)/2g} \equiv 0 \pmod p \quad\text{and}\quad (-c)^{(p+1)/2g} \equiv 1 \pmod p,$$

then $k(H,p) = 2(p+1)/g$.

The proofs for Theorems g–j are left to the reader.

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Therefore,

(7) $$F(0,1) = [1,1,1,\cdots] = \frac{1+\sqrt{4+1}}{2}$$

or

(8) $$\lim_{\alpha\to\infty} \frac{I_{\alpha-1}(2a)}{I_\alpha(2a)} = \frac{1+\sqrt{5}}{2} = \phi \text{ (the "golden" ratio)}.$$

Expressing ϕ in this manner as the limit of a ratio of modified Bessel Functions appears to be new [2].

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