THE FIBONACCI SERIES AND THE PERIODIC TABLE OF ELEMENTS

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The ratio, 0.6180, of the short, *a,* to the long, *b,* part of a line, divided so that $a/b = b/a + b$, is common in nature; often is called the "golden mean." An analogous line for the chemical elements is the distance between the centers of atoms in a compound. The alkali metal halide salts, which form from atoms at the extremes of reactivity in each series of the periodic table, should serve as reference compounds. If the short, *a*, part of the line is the covalent radius of the halogen atom, *X,* and the long part, *b,* is the corresponding radius of the alkali metal atom, *M,* in the same series, the mean of the ratio, *X/M,* for the five series is 0.605 ± 0.043, an understandable variation of 7% within itself as Table 1 shows. This mean is within 2.2% of the golden mean and possibly should be the same within experimental error. Only the covalent radii (see the Table) give this result. Calculations based on the ionic radii show a ratio as high as 2.27 and a 36% decrease from the first to the fourth series. Data are lacking for calculations based on the atomic radii, but in the present case the atoms in a compound, not the separate atoms, are under consideration.

Table 1
Covalent Radii of the Halide Salts and Calculation of the Factor, *R,* in the Fibonacci Equation

Column 1	2		3	4		5	
Halide pair	Observed* covalent radus, Å		Ratio *X/M*	Summation of observed radii		Correction Factor, *R,* Ratio Obs./Sum.	
	X	*M*		*X*	*M*	*X*	*M*
FLi	0.72	1.23	0.585				
ClNa	0.99	1.54	0.643	2.26	1.95	0.44	0.79
BrK	1.14	2.03	0.562	3.02	2.53	0.38	0.80
IRb	1.33	2.16	0.616	3.30	3.17	0.40	0.68
AtCs	1.45	2.35	0.618	3.68	3.49	0.39	0.67
?Fr					3.80		
Avg. or Min.		Avg.	0.605		Min.	0.39	0.67
Theory			0.618				
	Calculated†						
? Fr	1,56	2,55	0.610				

*Data are from the Sargent-Welch Company table commonly used by students.
†Calculated for the unknown francium halide as described in the text.

To approximate the position of the periodic table in the Fibonacci series we first use the lengths in angstrom units, Å, of the lithium and fluorine covalent radii for the construction in Fig. 1 of the smallest rectangle with dimensions of $a + b$ by b. From that rectangle, and the b by b square, larger and still larger rectangles and squares can be constructed in the usual manner [1, 3]. The centers of each square are marked with an alkali metal in increasing order, Li to Fr. Thus the pattern for a Fibonacci series is evident. A curved line, rather than the straight lines shown, could connect the centers between which the symbols for the other elements could be written (not done here for lack of space). In that way the periodic table would appear as a spiral, analogous to other spirals [1], the galaxies, the whirls in some flowers and plants, the horns of some animals, and the spirals in shells, all called the "golden horn." A simple calculation back to zero angstrom units suggests that the first series in the periodic table is roughly at the ninth number in the Fi bonacci series.

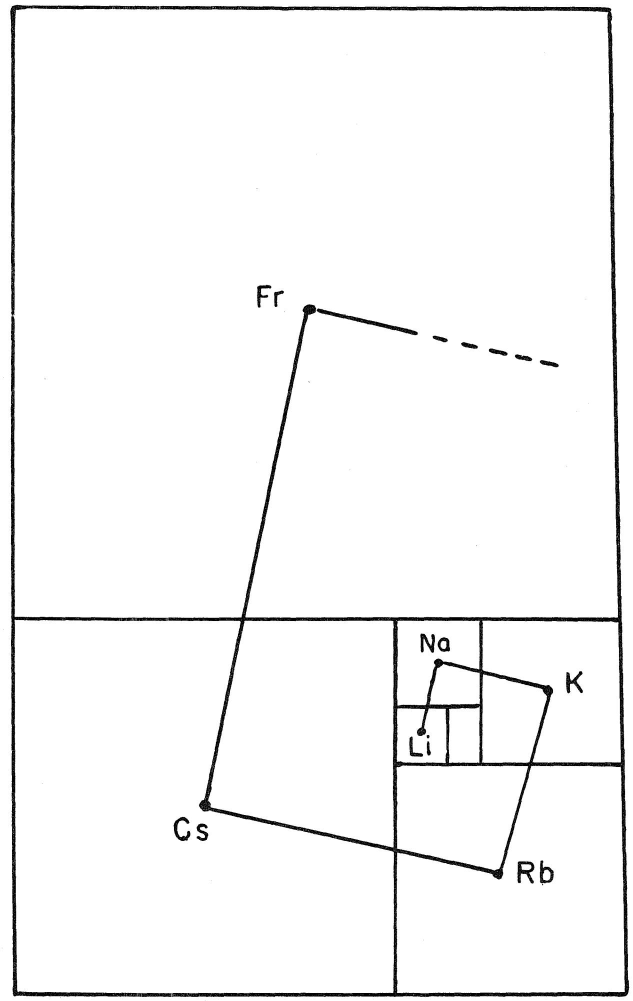

Fig. 1 Geometrical Arrangement According to the Fibonacci Pattern of the Five-Plus Series in the Periodic Table

The numbers in Column 2 of the Table, however, do not follow exactly the simple Fibonacci way where each succeeding number is the sum of the two preceding ones as in

$$U_n = U_{n-1} + U_{n-2}.$$

This situation is seen in Column 4, where the summation for chloride is 2.26 instead of the observed 0.99; and that for sodium is 1.95 instead of 1.54. Such abnormality probably results from the fact that the line (the sum of the two radii) does not pass through uniformly similar territory, for the specific volume of the halogen is much less and the atomic weight is much more than for the metal of each pair. To compensate for this situation the ratios of the observed to the summation for each radius are recorded in Column 5 for the X and M component of each pair. These values appear to attain minima-- 0.39 for the halogen and 0.67 for the metal. In other words in the formula $U_n = R(U_{n-1} + U_{n-2})$ the value of R is 0.39 when U_n is a halogen and 0.67 when a metal.

With these ratios, the value for the unknown halogen which would be paired with francium can be estimated as 1.56, and for francium would be 2.55. Then the ratio, *X/M* for that undiscovered salt would be 0.610,within 1.3% of the golden mean.

Whether that unknown halogen will ever be prepared may be doubted. Its atomic number would be 117 if the number of elements in the sixth series is the same as in the fifth. Wlodorski [4] has used the Fibonacci series to estimate the limiting stability of the nucleus in the transuranium elements and has concluded that efforts [5] to extend the series above number 114 cannot succeed. No objection to that prediction is here intended. However, attention should be drawn to a recent paper by Anders and co-workers [6] about the possibility of elements 115 (or 114, 113) having been found in a meterorite.

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