$$\sum_{i=0}^{[(n+1)/2]} \binom{n-i}{i}_r = F_{n+1}$$

where

$$\binom{n-i}{i}_r$$

is the polynomial coefficient in the i^{th} column and $(n-i)^{st}$ row of the left-adjusted array.

REFERENCES

1. N. B. Conkwright, *Introduction to the Theory of Equations,* Ginn and Co., 1941, Chapter X.
2. E-2487, Elementary Problem Section, *Amer. Math. Monthly,* Aug.–Sept., 1974.
3. Trudy Tong, "Some Properties of the Tribonacci Sequence and the Special Lucas Sequence," San Jose State University, Master's Thesis, 1970.
4. I. J. Schwatt, *Operations with Series,* Chelsea Pub. Co., New York, N.Y. (Reprint of 1924 book).
5. V. E. Hoggatt, Jr., and Marjorie Bicknell, "Multisection of the Fibonacci Convolution Array and Generalized Lucas Sequences," *The Fibonacci Quarterly,* to appear.
6. John Riordan, *An Introduction to Combinatorial Analysis,* Wiley & Sons, Inc., Pub., N.Y., 1958. Problem 27, p. 47.
7. V. E. Hoggatt, Jr., and Marjorie Bicknell, "Diagonal Sums of Generalized Pascal Triangles," *The Fibonacci Quarterly,* Vol. 7, No. 4, Nov. 1969, pp. 341–358.
8. V. E. Hoggatt, Jr., and Marjorie Bicknell, "Diagonal Sums of the Trinomial Triangle," *The Fibonacci Quarterly,* Vol. 12, No. 1, Feb. 1974, pp. 47–50.

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[Continued from p. 122.]

From this we have that

$$L(F(n)) = \frac{f(n+1) - (-1)^{F(n+2)} f(n-2)}{f(n-1)} \tag{3}$$

Now, letting $a = F(n)$, $b = F(n+1)$ in (2), we have

$$5f(n)f(n+1) = L(F(n+2)) - (-1)^{F(n)} L(F(n-1)). \tag{4}$$

Finally, substituting (3) for each term on the right of (4) and rearranging gives the required recursion.
It is interesting to note that a 5^{th} order recursion for $f(n)$ exists, but it is much more complicated.

Proposition.

$$f(n) = \frac{(5f(n-2)^2 + 2(-1)^{F(n+1)})f(n-3)^2 f(n-4) + f(n-2)(f(n-2) - (-1)^{F(n-1)} f(n-5))(f(n-1) - (-1)^{F(n)} f(n-4))}{2f(n-4)f(n-3)}$$

Proof. Use Equation (2) and the identity

$$L(a)L(b) = L(a+b) + (-1)^a L(b-a), \tag{5}$$

to obtain

$$5f(n)f(n+1) = 2L(F(n+2)) - L(F(n))L(F(n+1)).$$

Using (3) on the right-hand side and rearranging gives the required recursion.

REFERENCES

1. R. E. Whitney, "Composition of Recursive Formulae," *The Fibonacci Quarterly,* Vol. 4, No. 4, pp. 363.
2. E. A. Parberry, "On Primes and Pseudo-Primes Related to the Fibonacci Sequence," *The Fibonacci Quarterly,* Vol. 8, No. 1, pp. 52.

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