\[ M(n,k)/2^{n-1} = M(n-1, k)/2^{n-2} + kM(n-2, k)/2^{n-3}. \]

As \( M(1,k) = 1 \) and \( M(2,k) = 2 \) one can use induction to prove that \( M(n,k) \) is divisible by \( 2^{n-1} \).

Also solved by David G. Beverage, Wray G. Brady, Paul S. Bruckman, Herta T. Freitag, David Zeitlin, and the Proposer.

**OPERATIONAL IDENTITY**

B-339 Proposed by Gregory Wulczyn, Bucknell University, Lewisburg, Pennsylvania.

Establish the validity of E. Cesàro’s symbolic Fibonacci-Lucas identity \((2n+1)F_k = u^{2n+1} \); after the binomial expansion has been performed, the powers of \( u \) are used as either Fibonacci or Lucas subscripts. (For example, when \( n = 2 \) one has both \( 4F_2 + 4F_1 + F_0 = F_6 \) and \( 4L_2 + 4L_1 + L_0 = L_6 \).)

Solution by Graham Lord, Université Laval, Québec, Canada.

For a fixed \( K \) since both
\[ F_K a + F_{K-1} = a^K \quad \text{and} \quad F_K b + F_{K-1} = b^K, \]
the \( n^\text{th} \) power of each when added (algebraically) will give the result
\[ (F_K a + F_{K-1})^n = u^K n. \]

The desired equation is the special case when \( K = 3 \).


(Continued from page 284.)

**Solution by David Beverage, San Diego Community College, San Diego, California.**

By using the polynomials \( P_{2n+1}(x) \) * expressed explicitly as
\[
P_{2n+1}(x) = \sum_{r=0}^{n} 5^{n-r} (-1)^r \frac{(2n+1)![(2n-r)!)^r}{r!(2n+1-2r)!} x^{2n+1-2r} \]

and selecting \( m = 2n + 1 \), obtain
\[
Q = \frac{F_{mp}}{F_p} = F_p \cdot H \pm m,
\]
where \( H \) is a polynomial in \( F_p \).

Clearly,
\[ (F_p, m) \mid (F_p, Q). \]

Select \( m > 1 \) with integral coefficients and \( m \mid F_p (m \neq 0 (p)) \) in order that \( (F_p, Q) > 1 \). … The above conditions are satisfied for many numbers \( m \) and \( p \). One example: \( p = 7 \) and \( m = 13 \) produces
\[
\frac{F_{91}}{F_7} = 358465123875040793 = Q \quad \text{and} \quad (F_7, Q) = 13 > 1.
\]

Many other interesting divisor relationships may be obtained from the polynomials \( P_{2n+1}(x) \).

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