and

$$z(N) < C_\beta \beta^N$$

giving

$$\lim_{N \to \infty} \frac{z(N)}{Z(N)} = 0.$$

Corollary. On similar lines

$$\lim_{N \to \infty} \frac{Z(N)}{C_N} = \lim_{N \to \infty} \frac{z(N)}{C_N} = 0.$$

NOTE. Given a partition of N in terms of 1 and 2, if we rearrange the summands so as to get the maximum number of max we get a Z_2 composition. If we rearrange to get the maximum number of min we get a Z_1 composition. Roughly a Zeckendorf composition is either a maximax or a maximin composition.

REFERENCES

1. V. E. Hoggatt, Jr., and Krishnaswami Alladi, "Compositions and Recurrence Relations," *The Fibonacci Quarterly,* Vol. 13, No. 3 (Oct. 1975), pp. 233–235.
2. V. E. Hoggatt, Jr., and Krishnaswami Alladi, "Limiting Ratios of Convolved Recursive Sequences," *The Fibonacci Quarterly,* Vol. 15, No. 3 (Oct. 1977), pp. 211-214.

****

A TOPOLOGICAL PROOF OF A WELL KNOWN FACT ABOUT FIBONACCI NUMBERS

ETHAN D. BOLKER
Bryn Mawr College, Bryn Mawr, Pennsylvania

Theorem. Let p be a prime. Then there is a sequence $\{m_j\}$ of positive integers such that

$$F_{m_j} \equiv 1 - F_{m_j-1} \equiv 1 - F_{m_j+1} \equiv 0 \pmod{p^j}.$$

The proof depends on the following lemma.

Lemma. Let G be a topological group whose completion (in the natural uniformity) is compact. Let $g \in G$. Then the sequence $g, g^2, g^3, \cdots$ has a subsequence which converges to 1.

Proof. The sequence of powers of g has an accumulation point $h = \lim_{j \to \infty} g^{n_j}$ in the compact completion $\overline{G}$ of G. Let $m_j = n_{j+1} - n_j$. Then $g^{m_j} \to 1$ in $\overline{G}$ and hence in G.

To prove the theorem we shall apply the lemma to

$$g = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

in the group G of 2×2 integer matrices of determinant ± 1 topologized p-adically. That is, for every integer n write $n = p^k m$, $(p,m) = 1$ and set $\|n\|_p = p^{-k}$. Then for $A, B \in G$ let

$$d(A,B) = \max \left\{ \|A_{ij} - B_{ij}\|_p : i, j = 1, 2 \right\}$$

G equipped with the metric d satisfies the hypotheses of the lemma.

It is easy to check inductively that

$$g^m = \begin{pmatrix} F_{m+1} & F_m \\ F_m & F_{m-1} \end{pmatrix}.$$

[Continued on p. 280.]