Indeed, if $n = 1$, theorem is evident. Let us assume that the theorem is correct for $n \leqslant m$. The numbers of segment $[1, F_{2m+2} - 2]$ may be represented for part (1) of the theorem, as a sum of $\leqslant m$ Fibonacci numbers. Number $(F_{2m+2} - 2) + 1 = F_{2m+2} - 1$ may be represented for part (2) as a sum of $m + 1$ Fibonacci numbers. Number $(F_{2m+2} - 2) + 2 = F_{2m+2}$ is a Fibonacci number. The numbers of segment

$$[F_{2m+2} + 1, \quad F_{2m+2} + (F_{2m+1} - 1)] \tag{3}$$

are sums of number F_{2m+2} and of the corresponding numbers of segment $[1, F_{2m+1} - 1]$, which for part (1) of the theorem (since $F_{2m+1} - 1 \leqslant F_{2m+2} - 2$) are representable as a sum of $\leqslant m$ Fibonacci numbers. Number $F_{2m+2} + (F_{2m+1} - 1) + 1 = F_{2m+3}$ is a Fibonacci number. The numbers of the segment

$$[F_{2m+3} + 1, \quad F_{2m+3} + (F_{2m+2} - 2)]$$

are representable as a sum of $\leqslant m + 1$ Fibonacci numbers for the same reason as for the numbers of segment (3); though in this case we have the number F_{2m+3} and not F_{2m+2}. Thus all numbers not greater than

$$F_{2m+3} + (F_{2m+2} - 2) = F_{2(m+1)+2} - 2$$

are representable as sums of $\leqslant m + 1$ Fibonacci numbers. A correct decomposition of numbers $F_{2m+2} - 2$ and $F_{2m+2} - 1$ contains respectively (on the basis of the inductive assumptions) m and $m + 1$ terms. If to these decompositions we add on the left-hand side the term F_{2m+3} we obtain the correct decomposition of numbers $F_{2m+4} - 2$ and $F_{2m+4} - 1$. These latter contain respectively $m + 1$ and $m + 2$ terms. From this and from the theorem of Zeckendorf it follows that numbers $F_{2(m+1)+2} - 2$ and $F_{2(m+1)+2} - 1$ may be represented respectively as the sums of $m + 1$ (but not less) and respectively $m + 2$ (but not less) Fibonacci numbers.

By the way, it is clear that

$$F_{2n+2} - 2 = \sum_{i=1}^{2n} F_i = \sum_{i=1}^{n} F_{2i+1} .$$

One of more detailed works on these problems is [2].

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1. E. Zeckendorf, "Representation des nombres naturels par une somme de nombres de Fibonacci ou des nombres de Lucas," *Bull. Soc. Royale Sci. de Liege,* 3–9, 1972, pp. 779–182.
2. L. Carlitz, V. E. Hoggatt, Jr., and R. Scoville, "Fibonacci Representations," *The Fibonacci Quarterly,* Vol. 10, No. 1 (Special Issue, January 1972), pp. 1–28.

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LETTER TO THE EDITOR

April 28, 1970

In regard to the two articles, "A Shorter Proof" by Irving Adler (December, 1969 *Fibonacci Quarterly*) and "1967 as the Sum of Three Squares," by Brother Alfred Brousseau (April, 1967 *Fibonacci Quarterly*), the general result is as follows:

$$x^2 + y^2 + z^2 = n$$

is solvable if and only if n is not of the form $4^t(8k + 7)$, for $t = 0, 1, 2, \cdots, k = 0, 1, 2, \cdots$. See [1].

Since 1967 = 8(245) + 7, $1967 \neq x^2 + y^2 + z^2$. A lesser result known to Fermat and proven by Descartes is that no integer $8k + 7$ is the sum of three rational squares [2]. The *really* short and usual proof is:

For x, y, and z any integers, $x^2 \equiv 0$, 1, or 4 (mod 8) so that $x^2 + y^2 + z^2 \equiv 0, 1, 2, 3, 4, 5,$ or 6 (mod 8) or $x^2 + y^2 + z^2 \not\equiv 7 \pmod 8$.

REFERENCES

1. William H. Leveque, *Topics in Number Theory,* Vol. I, p. 133.
2. Leonard E. Dickson, *History of the Theory of Numbers,* Vol. II, Chap. VII, p. 259.

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