# GENERALIZED QUATERNIONS OF HIGHER ORDER

(44) 
$$= \Omega^{\lambda} W_{m+r+s} \Omega^{\mu} W_{n-r+t} + \epsilon q^{n-r} U_{2r-t+s-1} \Delta^{\lambda} \Omega^{\mu} U_{n-m-1}$$

(45) 
$$= W_{m+r+s} \Omega^{\Lambda+\mu} W_{n-r+t} + \epsilon q^{n-r} \Delta^{\Lambda} U_{2r-t+s-1} \Omega^{\mu} U_{n-m-1} .$$

Putting  $\lambda = 1$  and  $\mu = 1$  in (36), (39) and (40) gives us, respectively, (13), (26) and (27) of [2], while letting  $\lambda = 1$ ,  $\mu = 2$  in (39) and (40) gives, respectively, 28(a) and (b). If, however, we let t = 0, s = 0,  $\lambda = 1$  and  $\mu = 1$  in (43) we have as a special case result (29) of [2].

### REFERENCES

1. A. F. Horadam, "Basic Properties of a Certain Generalized Sequence of Numbers," *The Fibonacci Quarterly*, Vol. 3 (1965), No. 3, pp. 161-175.

 A.L. lakin, "Generalized Quaternions with Quaternion Components," The Fibonacci Quarterly, 1974 preprint.

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# LETTER TO THE EDITOR

### 16 September 1977

## Dear Professor Hoggatt:

(2)

In a recent article with Claudia Smith (*The Fibonacci Quarterly*, Vol. 14, No. 4, p. 343), you referred to the question whether a prime p and its square  $p^2$  can have the same rank of apparition in the Fibonacci sequence, and mentioned that Wall (1960) had tested primes up to 10,000 and not found any with this property.

I have recently extended this search and found that no prime up to 1,000,000 (one million) has this property. My computations in fact test the Lucas sequence for the property

(1) 
$$L_p \equiv 1 \pmod{p^2}$$
  $p = prime$ .

For  $\rho > 5$  this is easily shown to be a necessary and sufficient condition for  $\rho$  and  $\rho^2$  to have the same rank of apparition in the Fibonacci sequence, because of the identity

$$(L_p - 1)(L_p + 1) = 5F_{p-1}F_{p+1}.$$

So far I have shown that the congruence (1) does not hold for any prime less than one million; I hope to extend the search further at a later date.

You may wish to publish these results in *The Fibonacci Quarterly*.

Yours sincerely,

s/ Dr. L. A. G. Dresel The University of Reading, Berks, UK