so that if \( y = u_4 + s \) then
\[
1 - \frac{a^{-2}}{2} < -s(a^2u_2 - y) < 1 - \frac{a^{-2}}{2}.
\]
Since
\[
1 - \frac{a^{-2}}{2} > 0 \quad \text{and} \quad 1 - \frac{a^{-2}}{2} = \frac{a}{2}
\]

it follows that
\[
|a^2u_2 - y| < \frac{a}{2}.
\]
If there were an integer \( w \) such that \( |a^2w - y| < \frac{1}{2} \) it would follow that
\[
a^2|u_2 - w| < \frac{1 + a}{2} = \frac{a^2}{2}
\]
implying that \( w = u_2 \) and that \( y = u_4 \), contradicting the fact that \( |y - u_4| = 1 \). On the other hand, there is an integer \( x = y - u_2 \) such that \( |ax - y| < \frac{1}{2} \) since
\[
|ax - y| = |(a - 1)y - au_2| = (a - 1)|y - a^2u_2| < \frac{a(a - 1)}{2} = \frac{1}{2}.
\]
The existence of \( x \) (and the non-existence of \( w \)) satisfying these conditions, implies that \( y = v_2 \) for some \( v \in S \).

Thus,
\[
|a^2u_2 - v_2| < \frac{a}{2}.
\]
We now find
\[
u_2 - v_0 = |u_2 + v_1 - v_2| < |v_2a^{-2} - u_2| + |v_2(1 - a^{-2}) - v_1| = a^{-2}|v_2 - a^2u_2| + |v_2a - a^2v_1| < \frac{a^{-2}}{2} + \frac{a^{-1}}{2} = a^{-1} < 1
\]

so that \( u_2 = v_0 \in S_0 \).

Combining the results of Lemmas 3, 4, 5 we have
Theorem.
\[ S_0 = S_1 \cup S_2. \]

A GOLDEN DOUBLE CROSTIC: SOLUTION

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"Geometry has two great treasures: one is the theorem of Pythagoras; the other, the division of a line into extreme and mean ratio. The first we may compare to a measure of gold; the second we may name a precious jewel." J. Kepler. Quotation given in The Divine Proportion by Huntley (Dover, New York, 1970, p. 23).