REFERENCES

1. L. Carlitz & H. H. Ferns, "Some Fibonacci and Lucas Identities," *The Fibonacci Quarterly*, Vol. 8, No. 1 (1970), pp. 61-73.
2. V. E. Hoggatt, Jr., "Some Special Fibonacci and Lucas Generating Functions," *The Fibonacci Quarterly*, Vol. 9, No. 2 (1971), pp. 121-133.
3. J. W. Layman, "Certain General Binomial-Fibonacci Sums," *The Fibonacci Quarterly*, Vol. 15, No. 3 (1977), pp. 362-366.

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FIBONACCI CHROMOTOLOGY OR HOW TO PAINT YOUR RABBIT

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Readers of this journal are aware that Fibonacci numbers have been used to generate musical compositions [1], [2], and that the Golden Section ratio has appeared repeatedly in art and architecture. However, that Fibonacci numbers can be used to select colors in planning a painting is less well-known and certainly an exciting application.

One proceeds as follows, using a color wheel based upon the color theory of Johann Wolfgang von Goethe (1749-1832) and developed and extended by Fritz Faiss [3]. Construct a 24-color wheel by dividing a circle into 24 equal parts as in Figure 1. Let 1, 7, 13, and 19 be yellow, red, blue, and green, respectively. (In this system, green is both a primary color and a secondary color.) Halfway between yellow and red, place orange at 4, violet at 10, blue-green at 16, and yellow-green at 22. The other colors must proceed by even graduations of hue. For example, 2 and 3 are both a yellow-orange, but 2 is a yellow-yellow-orange, while 3 is a more orange shade of yellow-orange. The closest colors to use are: (You must also use your eye.)

1	Cadmium Yellow Light
2	Cadmium Yellow Medium
3	Cadmium Yellow Deep
4	Cadmium Orange or Vermilion Orange
5	Cadmium Red Light or Vermilion
6	Cadmium Red Medium
7	Cadmium Red Deep or Acra Red
8	Alizarin Crimson Golden or Acra Crimson
9	Rose Madder or Alizarin Crimson
10	Thalo Violet or Acra Violet
11	Cobalt Violet
12	Ultramarine Violet or Permanent Mauve or Dioxine Purple
13	Ultramarine Blue
14	French Ultramarine or Cobalt Blue
15	Prussian Blue
16	Thalo Blue or Phthalocyanine Blue or Cerulean Blue or Manganese Blue
17	Thalo Blue + Thalo Green
18	Thalo Green + Thalo Blue
19	Thalo Green or Phthalocyanine Green
20	Viridian

21 Emerald Green
22 Permanent Green
23 Permanent Green Light
24 Permanent Green Light + Cadmium Yellow Light

(Note: Expect problems in mixing a true tertiary color if using acrylic paints.)

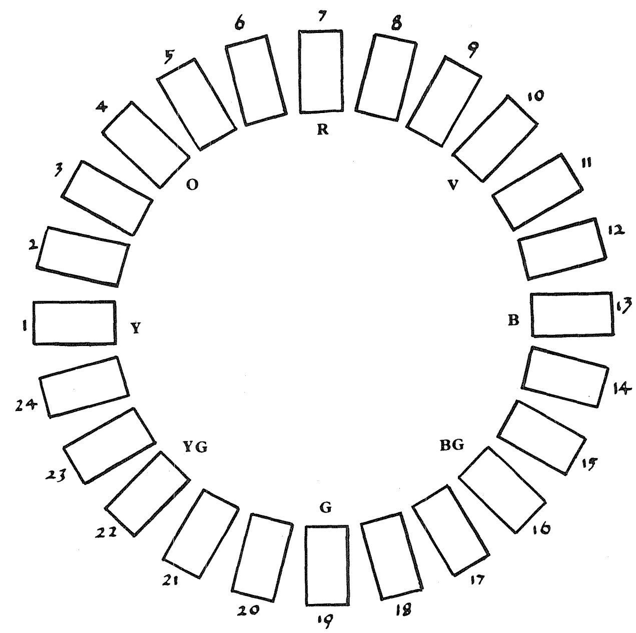

Fig. 1. 24-Color Wheel

To select colors to plan your painting, construct a second 24-color wheel but rather than coloring the spaces, cut out the spaces marked 1, 2, 3, 5, 8, 13, and 21. Place 1 at any position (primary or secondary color preferred) and use the colors thus exposed. The color under 1 should dominate, and 21 would be an accent color. This scheme solves the problem of color selection which occurs if one wishes to paint using bright, clear color; if one is accustomed to painting with "muddy" colors, he may feel that he has no problems with harmony.

Fritz Faiss has many other color schemes based upon the 24-color wheel. The color sequences based upon the Fibonacci sequence are particularly pleasing, and Fritz Faiss has done many paintings using these color sequences. Unfortunately, to fully appreciate the beauty of the color combinations that arise, one needs to actually see a properly constructed color wheel and some examples of its application.

All the color schemes generated as just described are quite lovely, and the Lucas sequence also seems to select pleasant schemes or, at least, nondiscordant ones. But, to see what a color battlefield can be constructed, make a 24-color wheel using the more familiar yellow, red, and blue as primary colors placed at 1, 9, and 17 with the in-between colors again placed in order by hue (so that, for example, 21 is green and 19 is blue-green, 20 is a green blue-green, and 18 is halfway between blue and blue-green). Then, the Fibonacci sequence does not select pleasing combinations, and one comes to appreciate the problem involved in selecting bright, true colors which harmonize.

This short article certainly will pose more questions than it answers, since mathematicians are not usually accustomed to thinking about color theory as used in painting; Fritz Faiss has devoted fifty years to the study of color theory in art. Fibonacci numbers seem to form a link from art to music; perhaps some creative person will compose a Fibonacci ballet, or harmonize Fibonacci color schemes with Fibonacci music.

REFERENCES

1. Hugo Norden, "Per Nørgård's 'Canon,'" *The Fibonacci Quarterly*, Vol. 14, No. 2 (April 1976), pp. 125-128.
2. Hugo Norden, "Proportions and the Composer," *The Fibonacci Quarterly*, Vol. 10, No. 3 (April 1972), pp. 319-323.
3. Fritz Faiss, *Concerning the Way of Color: An Artist's Approach* (Valencia Hills, California: The Green Hut Press, 1972).

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ON THE DENSITY OF THE IMAGE SETS OF CERTAIN ARITHMETIC FUNCTIONS—II

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1. INTRODUCTION

Throughout this article, we will be using the following notation: $n \geq 0$ is an arbitrary nonnegative integer and $n = \sum_{j=0}^{k} d_j b^j$ its representation as an integer in base b, $b \geq 2$ arbitrary. Define

$$T(n) = n + \sum_{j=0}^{k} d_j \qquad [T(0) = 0] \tag{1.1}$$

$$\mathcal{R} = \{n \mid n = T(x) \text{ for some } x\} \quad \text{and}$$

$$\mathcal{C} = \{n \mid n \neq T(x) \text{ for any } x\}.$$

It has been shown ([1]) that the set $\mathcal{C}$ is infinite for any base b. More generally, it is true that $\mathcal{C}$ has asymptotic density and that $\mathcal{C}$ is a set of positive density; these results are derived from the following more general theorem and its corollary (proofs of which may be found in [2]).

Theorem: Let

$$n = \sum_{j=0}^{k} d_j b^j, \; b \geq 2 \text{ arbitrary},$$

and define

$$T(n) = n + \sum_{j=0}^{k} f(d_j,\, j) \quad \text{and} \quad \mathcal{R} = \{n \mid n = T(x) \text{ for some } x\},$$