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A PRIMER ON STERN'S DIATOMIC SEQUENCE

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PART III: ADDITIONAL RESULTS

An examination of the sequence yields corollaries to some of the previously known results. Being fundamentally Fibonacci minded, and at the onset not aware of the works of Stern, Eisenstein, Lehmer and Lind, we noticed the following results not already mentioned—some may even seem trivial.

(1) $s(n,1) = n$
$s(n,2) = n - 1$
$s(n,4) = n - 2$
$\vdots$
$s(n,2^m) = n - m$

(2) $s(n,a2^m) = s(n - m,a)$

(3) Another statement of symmetry is $s(n,2^{n-2} - a) = s(n,2^{n-2} + a)$

(4) $s(n,2^{n-1}) = 1$
$s(n,2^{n-2}) = 2$
$\vdots$
$s(n,2^{n-k}) = k$

(5) $s\left(n + k - 1, \dfrac{2^k - (-1)^k}{3}\right) = F_{k-1} + nF_k$, or

$$s\left(N, \frac{2^k - (-1)^k}{3}\right) = F_{k-1} + (N - k + 1)F_r$$

(6) $s\left(n, \dfrac{2^{n-2} + (-1)^{n-1}}{3}\right) = L_{n-1}$ when L_n are Lucas numbers

(7) $s(n,2^k) + s(n,2^{k+1}) = s(n,3 \cdot 2^{k-1})$

(8) $s(n,K \cdot 2^{k-1}) = s(n,K \cdot 2^k) + 1$

(9) $s(n,3 \cdot 2^{m-1}) = 2(n - m) + 1, \quad n > 0$

$s(n,5 \cdot 2^{m-1}) = 3(n - m) - 1, \quad n > 1$

$s(n,7 \cdot 2^{m-1}) = 3(n - m) - 2, \quad n > 1$

$s(n,9 \cdot 2^{m-1}) = 4(n - m) - 5, \quad n > 2$

$s(n,11 \cdot 2^{m-1}) = 5(n - m) - 7, \quad n > 2$

$s(n,13 \cdot 2^{m-1}) = 5(n - m) - 8, \quad n > 2$

$s(n,15 \cdot 2^{m-1}) = 4(n - m) - 7, \quad n > 2$

etc.

where $m = 1, 2, 3, \ldots$

(10) The table on page 320 is the sequence of combinatorial coefficients mod 2. Hoggatt informed us that he suspected the sums of the rising diagonals were Stern numbers—he was right.

The formal statement of the problem is that

$$\sum_{i-0}^{\left[\frac{j-1}{2}\right]} \binom{j - i - 1}{i} \bmod 2 = s(k + 1, j - 2^k)$$

where $k = [\log_2 j]$ and $2^k \le j \le 2^{k+1}$.

The proof is by induction relying essentially on the following theorem.

Theorem: Given the binomial coefficient mod 2, $\begin{matrix} n \\ k \end{matrix}_2$, then

$$\binom{n}{k}_2 \equiv \binom{2n + 1}{2k} + \binom{2n}{2k + 1} \bmod 2$$

the right-hand side (after some reduction) may be rewritten as

$$\frac{(2n)(2n - 2) \ldots (2n - 2k + 2)}{(2k)(2k - 2) \ldots 4 \cdot 2} \cdot \frac{(2n - 1)(2n - 3) \ldots (2n - 2k + 1)}{(2k - 1) \ldots 5 \cdot 3 \cdot 1}.$$

The right-hand factor is $\equiv 1 \bmod 2$; therefore, this is congruent to

$$\frac{2^k \cdot n(n - 1) \ldots (n - k + 1)}{2^k \cdot k(k - 1) \ldots 2 \cdot 1}$$

which is congruent to

$$\binom{n}{k}.$$

STERN NUMBERS VERSUS SUMS OF RISING DIAGONALS OF BINOMIAL NUMBERS MOD 2

	Column																			
Row	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
0	1																			$1 = S[1,0]$
1	1	1																		$1 = S[2,0]$
2	1	0	1																	$2 = S[2,1]$
3	1	1	1	1																$1 = S[3,0]$
4	1	0	0	0	1															$3 = S[3,1]$
5	1	1	0	0	1	1														$2 = S[3,2]$
6	1	0	1	0	1	0	1													$3 = S[3,3]$
7	1	1	1	1	1	1	1	1												$1 = S[4,0]$
8	1	0	0	0	0	0	0	0	1											$4 = S[4,1]$
9	1	1	0	0	0	0	0	0	1	1										$3 = S[4,2]$
10	1	0	1	0	0	0	0	0	1	0	1									$5 = S[4,3]$
11	1	1	1	1	0	0	0	0	1	1	1	1								$2 = S[4,4]$
12	1	0	0	0	1	0	0	0	1	0	0	0	1							$5 = S[4,5]$
13	1	1	0	0	1	1	0	0	1	1	0	0	1	1						$3 = S[4,6]$
14	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1					$4 = S[4,7]$
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1				$1 = S[5,0]$
16	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1			$5 = S[5,1]$
17	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1		$4 = S[5,2]$
18	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	$7 = S[5,3]$

Many thanks go to Dudley [1] and to Hoggatt for sponsoring the authors to write this series of articles. At this writing, the authors still do not know the general form of

$$s\left(n, (2r + 1)2^m\right)$$

and suggest that some ambitious reader show the relationship to the Fibonacci numbers.

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