5. Verner E. Hoggatt, Jr., & Marjorie Bicknell-Johnson. "Generalized Fibonacci Numbers Satisfying $u_{n+1}u_{n-1} - u_n^2 = \pm 1$." *The Fibonacci Quarterly* 16, No. 2 (1978):130-137.
6. Serge Lang. *Algebraic Number Theory*. Reading, Mass.: Addison-Wesley Publishing Company, 1970. P. 65.

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LETTER TO THE EDITOR

DAVID L. RUSSELL
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Dear Professor Hoggatt:

. . . In response to your request for me to point out the errors in your article "A Note on the Summation of Squares," *The Fibonacci Quarterly* 15, No. 4 (1977):367-369, . . . I have enclosed a xerox copy of your paper with corrections marked. The substantive errors occur in the top two equations of p. 369, where an incorrect sign and some minor errors result in an incorrect denominator for the RHS. As an example, consider the case $p = 1$, $q = 2$, $n = 4$; your formula evaluates to 0, which is clearly incorrect:

$$P_0 = 0,\ P_1 = 1,\ P_2 = 1,\ P_3 = 3,\ P_4 = 5,\ P_5 = 11,\ P_6 = 21;$$
$$8P_5P_4 - (P_6^2 - 1) = (8)(11)(5) - 440 = 0.$$

Only if the denominator is also zero does a numerator of zero make sense.

Sincerely yours,
[*David L. Russell*]

CORRECTIONS TO "A NOTE ON THE SUMMATION OF SQUARES" BY VERNER E. HOGGATT, JR.

The following corrections to the above article were noted by Prof. David L. Russell.

Page 368: The equation on line 19, $q^{n-1}P_2P_1 = q^{n-1}P_1^2 + q^nP_1P_0$, should be:

$$q^nP_2P_1 = q^nP_1^2 + q^{n+1}P_1P_0$$

The equation on line 27, $P_{j+2}^2 = P^2P_{j+1}^2 + q^2P_j^2 + 2pqP_jP_{j+1}$, should be:

$$P_{j+2}^2 = p^2P_{j+1}^2 + q^2P_j^2 + 2pqP_jP_{j+1}$$

In the partial equation on line 32 (last line) the = sign should be a - (minus) sign.

Page 369: Lines 1-11 should read:

$$pP_{n+1}^2 + \left(\sum_{j=1}^{n} P_j^2\right)\left(p + \frac{(1-q)(p^2+q^2-1)}{2pq}\right)$$
$$= P_{n+2}P_{n+1} + \frac{1-q}{2pq}[P_{n+2}^2 + P_{n+1}^2 - 1 - p^2P_{n+1}^2]$$

$$\sum_{j=1}^{n} P_j^2 = \frac{P_{n+2}P_{n+1} - pP_{n+1}^2 + \frac{(1-q)}{2pq}[P_{n+2}^2 + P_{n+1}^2(1-p^2) - 1]}{(2p^2q + p^2 + q^2 - 1 - qp^2 - q^3 + q)/2pq}$$

Testing $p = 1$, $q = 1$,

$$\sum_{i=1}^{n} F_i^2 = \frac{2F_{n+2}F_{n+1} - 2F_{n+1}^2}{2} = F_{n+1}F_n.$$

For $q = 1$ only,

$$\sum_{i=1}^{n} P_i^2 = \frac{2pP_{n+2}P_{n+1} - 2p^2P_{n+1}^2}{2p^2} = \frac{P_{n+2}P_{n+1} - pP_{n+1}^2}{p} = \frac{P_{n+1}P_n}{p}$$

so that

$$\sum_{i=1}^{n} P_i^2 = P_{n+1}P_n/p.$$

Thus,

$$\sum_{j=1}^{n} P_j^2 = \frac{p\left[2qP_{n+2}P_{n+1} - 2pqP_{n+1}^2 + \frac{(1-q)}{p}[P_{n+2}^2 + (1-p^2)P_{n+1}^2 - 1]\right]}{(q+1)(p^2 - (q-1)^2)}$$

$$= \frac{p\left[2q^2(P_{n+1}P_n) + \frac{(1-q)}{p}[P_{n+2}^2 + (1-p^2)P_{n+1}^2 - 1]\right]}{(q+1)(p^2 - (q-1)^2)}.$$

According to Prof. Russell, this last equation can also be written as

$$\sum_{j=1}^{n} P_j^2 = \frac{2pq^2P_{n+1}P_n + (1-q)[P_{n+2}^2 + (1-p^2)P_{n+1}^2 - 1]}{(q+1)(p^2 - (q-1)^2)}$$

$$= \left[\frac{2pqP_nP_{n+1} + (1-q)P_{n+1}^2 + q^2(1-q)P_n^2}{(q+1)(p^2 - (q-1)^2)}\right]_0^n,$$

since $P_{n+2}^2 = p^2P_{n+1}^2 + 2pqP_nP_{n+1} + q^2P_n^2$.

The author is grateful to Prof. Russell for the above corrections.

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