LETTER TO THE EDITOR

The Editor,
Fibonacci Quarterly.

Dear Dr. Hoggatt,

I refer to the article, "Dying Rabbit Problem Revived" in the December 1963 issue. The solution given there is patently wrong — if only because the alleged number of rabbits tends to minus infinity as n tend to infinity. It may easily be shown that the correct answer, X_n, is given by the recurrence relation

$$X_{n+13} = X_{n+12} + X_{n+11} - X_n \;, \quad n \quad 0$$

together with the initial conditions

$$X_n = F_{n+1} \quad \text{for } n = 0, 1, \ldots, 11; \quad X_{12} = 232 \;.$$

In view of the fact that the two equations $z^2 - z - 1 = 0$ and $z^{13} - z^{12} - z^{11} + 1 = 0$ have no common root, it is clear that the answer can never be expressed simply as a linear expression in Fibonacci and Lucas numbers whose coefficients are merely polynomials in n. For, any such expression, Y, where the highest power of n which occurs is n^m, satisfies

$$(E^2 - E - 1)^{m+1}\, Y = 0 \;.$$

In particular the expression found by Bro. Alfred satisfies

$$(E^2 - E - 1)^2\, Y = 0 \;.$$

The error made by Bro. Alfred stems from his table on p. 54 where the number of dying rabbits in the (n+13)th month is seen to be F_n for n = 1, 2, ... 11 and it is then assumed without proof that this is true for other values of n. In fact the very next but one value on n, namely n = 13 shows that this is false. In fact of course the number of dying rabbits in the (n+13)th month equals the number of bred rabbits in the (n+1)th month, and this will be less than F_n for all n exceeding 12.

Yours sincerely, (John H. E. Cohn)

BEDFORD COLLEGE
(University of London)