

PROPORTIONS IN MUSIC

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The systematic organization of a musical composition within a pre-determined time span by means of the lower numbers of the Fibonacci and Lucas series, singly or in combination, is common practice indeed. It seems that the more profound the composer, the stricter is his application of these proportions in the musical structure.

A neatly contrived example is found in the first fugue in The Art of the Fugue by Johann Sebastian Bach. The formal and thematic materials can be listed quite simply as follows:

number of measures = 78 (13 x 6)
 number of entries = 11

Of the 11 entries of the subject and answer, 9 begin on either "D" (the tonic) or "A" (the dominant), while 2 begin on "E". These 2 entries that begin on the note "E" define the form of the composition. The first, entry No. 8, occurs at measure 40, thereby beginning the latter half of the 78-measure time span; while the second, entry No. 9, comes at measure 49, thus announcing the start of the 5/13 portion of the 8/13 + 5/13 (48 + 30) division. This formally significant pair of entries is assigned to the Tenor and Soprano parts, respectively.

The total number of 11 entries, however, is distributed within the time span as follows:

<p style="text-align: center;">7</p> <p style="text-align: center;">before the middle (measures 1-39)</p> <p style="text-align: center;">3 : 4</p> <p style="text-align: center;">Answers : Subjects</p> <p style="text-align: center;">3 : 4</p> <p style="text-align: center;">begin on "D" begin on "A"</p>	<p style="text-align: center;">4</p> <p style="text-align: center;">after the middle (measures 40-78)</p> <p style="text-align: center;">1 : 3</p> <p style="text-align: center;">Subjects : Answers</p> <p style="text-align: center;">1 : 2</p> <p style="text-align: center;">begin on "A" begin on "E"</p>
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The fugue is given in full on the two following pages. Both the measures and entries are numbered and the type and starting note of each entry is indicated so that the reader can follow the plan of the composition. As several recordings of this music are available, it should be easy to experience this time span utilization audibly.

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**REMARKS ON A SECOND ORDER
RECURRING SEQUENCE**

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Among the second order recurring sequences, the degenerate sequence $U_n = n^2 - n - 1$ is of some interest. In fact, we can observe the following special property among the more unusual properties such sequences have:

$$U_n U_{n+1} = U_{n^2-1} .$$

Proof:

$$\begin{aligned} U_n U_{n+1} &= [n^2 - n - 1] [(n+1)^2 - (n+1) - 1] \\ &= (n^2 - n - 1)(n^2 + n - 1) \\ &= (n^2 - 1)^2 - n^2 \\ &= (n^2 - 1)^2 - (n^2 - 1) - 1 \\ &= U_{n^2-1} . \end{aligned}$$

In what way this property can be generalized remains to be seen.

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FUGA I

a 4 voci.

J. S. Bach.

Andante con moto.
sempre legato

① *p* Subject

② Answer

③ Subject

④ Answer

⑤ Subject

⑥ Subject

⑦ Answer

⑧ Answer

MIDDLE

This musical score is written for piano in a single system with two staves (treble and bass clef). The key signature has one sharp (F#) and the time signature is 3/4. The score is divided into several measures, with measure numbers 42 through 78 indicated. The music is characterized by intricate rhythmic patterns and dynamic markings.

Key features of the score include:

- Measure 42:** Starts with a *sf* (sforzando) dynamic.
- Measure 43:** Continues the melodic line.
- Measure 44:** A *cresc.* (crescendo) marking is present.
- Measure 47:** A *dim.* (diminuendo) marking is present.
- Measure 48:** A *f* (forte) dynamic marking is present.
- Measure 49:** A circled number '9' is placed above the staff, followed by the word "Answer".
- Measure 52:** A *sf* dynamic marking is present.
- Measure 53:** A *dim.* dynamic marking is present.
- Measure 54:** A circled number '10' is placed below the staff, followed by the word "Subject".
- Measure 55:** A *p* (piano) dynamic marking is present.
- Measure 56:** A *cresc.* dynamic marking is present.
- Measure 57:** A *dim.* dynamic marking is present.
- Measure 60:** A *cresc.* dynamic marking is present.
- Measure 62:** A *f* dynamic marking is present.
- Measure 65:** A *dim.* dynamic marking is present.
- Measure 67:** A *cresc.* dynamic marking is present.
- Measure 70:** A *f* dynamic marking is present.
- Measure 71:** A *fz* (forzando) dynamic marking is present.
- Measure 72:** A *fz* dynamic marking is present.
- Measure 73:** A *sf* dynamic marking is present.
- Measure 74:** A circled number '11' is placed below the staff, followed by the word "Answer".
- Measure 75:** A *sf* dynamic marking is present.
- Measure 76:** A *sf* dynamic marking is present.
- Measure 77:** A *sf* dynamic marking is present.
- Measure 78:** A *rallent. dim.* (rallentando, diminuendo) marking is present.