PROPORTIONS IN MUSIC

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The systematic organization of a musical composition within a pre-determined time span by means of the lower numbers of the Fibonacci and Lucas series, singly or in combination, is common practice indeed. It seems that the more profound the composer, the stricter is his application of these proportions in the musical structure.

A neatly contrived example is found in the first fugue in The Art of the Fugue by Johann Sebastian Bach. The formal and thematic materials can be listed quite simply as follows:

number of measures = 78 (13 x 6)
number of entries = 11

Of the 11 entries of the subject and answer, 9 begin on either "D" (the tonic) or "A" (the dominant), while 2 begin on "E". These 2 entries that begin on the note "E" define the form of the composition. The first, entry No. 8, occurs at measure 40, thereby beginning the latter half of the 78-measure time span; while the second, entry No. 9, comes at measure 49, thus announcing the start of the 5/13 portion of the 8/13 + 5/13 (48 + 30) division. This formally significant pair of entries is assigned to the Tenor and Soprano parts, respectively.

The total number of 11 entries, however, is distributed within the time span as follows:

7			4		
before the middle (measures 1-39)			after the middle (measures 40-78)		
3	:	4	1	:	3
Answers		Subjects	Subjects		Answers
3	:	4	1	:	2
begin on "D"		begin on "A"	begin on "A"		begin on "E"

The fugue is given in full on the two following pages. Both the measures and entries are numbered and the type and starting note of each entry is indicated so that the reader can follow the plan of the composition. As several recordings of this music are available, it should be easy to experience this time span utilization audibly.

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REMARKS ON A SECOND ORDER RECURRING SEQUENCE

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Among the second order recurring sequences, the degenerate sequence $U_n = n^2-n-1$ is of some interest. In fact, we can observe the following special property among the more unusual properties such sequences have:

$$U_n U_{n+1} = U_{n^2-1} .$$

Proof:

$$\begin{aligned} U_n U_{n+1} &= \left[n^2-n-1\right]\left[(n+1)^2-(n+1)-1\right] \\ &= (n^2-n-1)(n^2+n-1) \\ &= (n^2-1)^2-n^2 \\ &= (n^2-1)^2-(n^2-1)-1 \\ &= U_{n^2-1} . \end{aligned}$$

In what way this property can be generalized remains to be seen.

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FUGA I

a 4 voci.

Andante con moto. J. S. Bach.

sf
p
cresc.
dim.
f
sf
9
Answer
E
48+30
[8:5]
dim.
p
cresc.
10
D
Subject
dim.
p
cresc.
f
dim.
cresc.
f
f
fz
sf
11
Answer
sf
sf
sf
sf
sf
rallent. dim.
ff
p