EXPLORING GEOMETRIC-ALGEBRAIC FIBONACCI PATTERNS

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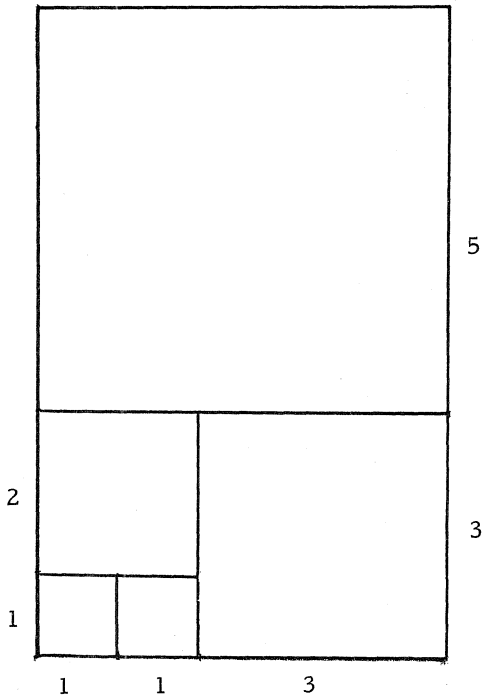
Probably one of the early experiences of most Fibonacci enthusiasts is becoming acquainted with the following geometric figure. Two squares of side 1 are placed next to each other horizontally. On top of them is constructed a square of side 2. Next to the resulting figure is located a square of side 3. On top of this a square of side 5 is adjoined. And so on. The demonstrator points out triumphantly that he has added together

$$F_1^2 + F_2^2 + F_3^2 + F_4^2 + F_5^2 \quad .$$

And what is the result? F_5F_6. This quickly leads to an intuitive result:

$$\sum_{i=1}^{n} F_i^2 = F_n F_{n+1}$$



Now I suppose many of us have asked ourselves the question: Where is example number two? Why don't we have more of this?

The problem may be stated as follows. We wish to find a geometric pattern involving Fibonacci numbers so that there will be a correlation between the geometry and some algebraic formula. In working along these lines, for example, a square F_n on each side can be filled with Fibonacci squares, the idea being to absorb the area with Fibonacci squares of as large a side as possible. For this purpose, a square of side F_{n-1} can be located in one corner; three squares of side F_{n-2} can then be placed in the remaining corners. This leaves as a balance two rectangles of dimensions F_{n-2} by F_{n-3}. Hence we have a formula

$$F_n^2 = F_{n-1}^2 + 3F_{n-2}^2 + 2 \sum_{k=1}^{n-3} F_k^2$$

which correlates with a geometric figure.

Here are additional suggestions though not all have been tried and hence there is no guarantee that results will be forthcoming.

(1) Covering Fibonacci rectangles, such as F_nF_{n-2}, F_nF_{n-3}, etc. by Fibonacci squares.

(2) Filling Lucas squares or rectangles with Fibonacci squares.

(3) Reversing the operation and using Lucas squares instead of Fibonacci squares.

(4) Finding space analogues using Fibonacci cubes.

Discoveries pertaining to this type of problem will be published in the April 1965 issue of the Fibonacci Quarterly. It would be advisable to have this material with the Editor by March 1st.

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