Also solved by Paul S. Bruckman, L. Kuipers, Bob Prielipp, Lawrence Somer, and the proposer.

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LETTER TO THE EDITOR

February 3, 1989

Dear Dr. Bergum,

I'd like to point out that some results which appeared in Michael Mays's recent article, "Iterating the Division Algorithm" [*Fib. Quart.* 25 (1987):204-213] were already known.

In particular, his Algorithm 6, which on input (b, a) sets $a = a$ and $a_1 = b \bmod a$, appeared in my paper, "Metric Theory of Pierce Expansions," [*Fib. Quart.* 24 (1986):22-40]. His Theorem 4, proving that $L(b, a) \le 2\sqrt{b} + 2$ [where $L(b, a)$ is the least n such that $a = 0$), appears in my paper as Theorem 19.

Let Ω, Ω' be defined as follows: we write $f(n) = \Omega(g(n))$ if there exist c, N such that $f(n) \ge cg(n)$ for all $n \ge N$. We write $f(n) = \Omega'(g(n))$ if there exists c such that $f(n) \ge cg(n)$ infinitely often. Since my paper appeared, I have proved

$$\max_{1 \le a \le n} L(n, a) = \Omega'(\log n)$$

and

$$\sum_{1 \le a \le n} L(n, a) = \Omega(n \log \log n).$$

The details are available to those interested.

Recently, I also stumbled across what may be the first reference to this type of algorithm. It is J. Binet, "Recherches sur la théorie des nombres entiers et sur la résolution de l'équation indéterminee du premier degré qui n'admet que des solutions entières," *J. Math. Pures Appl.* 6 (1841):449-494. Binet's algorithm, however, takes the *absolutely least residue* at each step, rather than the positive residue, and it is therefore easier to prove there are no long expansions.

Sincerely yours,

Jeffrey Shallit