$$\sum_{n=1}^{p-1}\binom{p}{n}\rho^{n} = 0, \quad \sum_{n=1}^{p-1}\binom{p}{n}\rho^{-n} = 0.$$

Multiplying the first equation with ρ^{k}, the second with ρ^{-k}, and using the easily verified formula

$$u_n = \frac{(-1)^{n-1}}{\sqrt{-3}}(\rho^{n} - \rho^{-n}),$$

we get

$$\sum_{n=1}^{p-1}(-1)^{n-1}u_{n+k}\binom{p}{n} = 0.$$

Dividing by p and using

$$\frac{1}{p}\binom{p}{n} \equiv \frac{(-1)^{n-1}}{n} \pmod{p}, \quad 1 \le n \le p-1,$$

we get the assertion.

Also solved by Paul S. Bruckman.

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(continued from page 288)

$Z_i(t)$ represents the number of zeros of f_t which are ε-close to η_i. By invariance of the complex integral, the functions $Z_i(t)$ are constant since the functions f_t vary continuously and do not vanish on the path of integration. Hence, $Z_i(0) = Z_i(1)$ for each i. This says that in a small neighborhood of each zero of f_1, there is a one-to-one correspondence of zeros of f_1 with zeros of f_0, in the required manner. $\square$

In the case of our given functions, we find that the zeros of the polynomial $f_n(z)$ are close to the zeros of $g_n(z)$, which lie on the circle $|z| = \alpha$, as required, and the zeros of f_n get closer to the circle as $n \to \infty$. $\blacksquare$

Also solved by P. Bruckman, O. Brugia & P. Filipponi, L. Kuipers, and the proposer.

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