EXPLORING GENERALIZED FIBONACCI-LUCAS RELATIONS

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A generalized Fibonacci sequence with positive terms can be formed by taking any two positive integers and then applying the law of formation of Fibonacci sequences which states that each term is the sum of the two preceding terms. As a further refinement, one might number the terms of the sequence according to the scheme set up in [1]. In this arrangement, if f_i is the term of a generalized Fibonacci sequence, then f_1 if characterized by the fact that $f_1 < f_2/2$. (Note. This manner of notation does NOT apply to the Fibonacci sequence: $1, 1, 2, 3, 5, 8 \ldots$ as usually numbered.) Then the characteristic number of the sequence which we have denoted D (see ref. 1) is given by:

$$D = f_1^2 - f_0 f_2$$

We now associated with this generalized Fibonacci sequence a Lucas sequence whose terms g_n are defined by:

$$g_n = f_{n-1} + f_{n+1}$$

It can be shown that this is also a Fibonacci sequence and that the characteristic number of the sequence is numerically equal to 5D.

A. F. Horadam has worked out and reported a large number of relations that apply to generalized Fibonacci sequences [2]. The present exploration is concerned with relations involving both f and g. A few samples are:

$$g_{2n}^2 - g_0^2 = 5 (f_{2n}^2 - f_0^2)$$

$$g_{n+1} g_{p+1} + g_n g_p = 5(f_{n+1} f_{p+1} + f_n f_p)$$

$$f_{2n+1} = F_n g_{n+1} + (-1)^n f_1$$

where F_n is a member of the Fibonacci sequence properly so-called.

We would urge readers to report any and all relations of the above type that they may find, whether their work is extensive and formal or whether it is in the nature of a particular note. Proofs of results are also in order, but their absence should not prevent reporting a known relation.

REFERENCES

(1)   Brother U. Alfred, On the Ordering of the Fibonacci Sequence, Fibonacci Quarterly, Dec., 1963, pp. 43-46.

(2)   A. F. Horadam, A Generalized Fibonacci Sequence, Amer. Math. Monthly, May 1961, pp. 455-459.

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