



# The Fibonacci Quarterly

THE OFFICIAL JOURNAL OF THE FIBONACCI ASSOCIATION

## TABLE OF CONTENTS

On the Square Roots of Triangular Numbers .....	<i>A. Behera and G. K. Panda</i>	98
Generalizations of Some Identities of Long .....	<i>Ray Melham</i>	106
Lucas Sequences and Functions of a 3-by-3 Matrix .....	<i>R. S. Melham</i>	111
Book Review: <i>The Golden Ratio and Fibonacci Numbers</i> , by Richard A. Dunlap .....	<i>Reviewed by Marjorie Bicknell-Johnson</i>	116
On the 2-Adic Valuations of the Truncated Polylogarithm Series .....	<i>Henri Cohen</i>	117
Algorithmic Summation of Reciprocals of Products of Fibonacci Numbers .....	<i>Stanley Rabinowitz</i>	122
Division of Fibonacci Numbers by $k$ .....	<i>Herta T. Freitag and Piero Filipponi</i>	128
Author and Title Index for Sale .....		134
The Girard-Waring Power Sum Formulas for Symmetric Functions, and Fibonacci Sequences .....	<i>Henry W. Gould</i>	135
A Generalization of Jacobsthal Polynomials .....	<i>M. N. S. Swamy</i>	141
Even Ducci-Sequences .....	<i>Anne L. Ludington-Young</i>	145
Eulerian Numbers: Inversion Formulas and Congruences Modulo a Prime .....	<i>J. E. Nymann and Ricardo A. Sáenz</i>	154
Algorithmic Manipulation of Second-Order Linear Recurrences .....	<i>Stanley Rabinowitz</i>	162
In Memoriam—Richard Spain Vine .....		177
The Golden Section and Newton Approximation .....	<i>Georg Johann Rieger</i>	178
Elementary Problems and Solutions .....	<i>Edited by Stanley Rabinowitz</i>	180
Advanced Problems and Solutions .....	<i>Edited by Raymond E. Whitney</i>	185
New Problem Web Site .....		192

VOLUME 37

MAY 1999

NUMBER 2

## PURPOSE

The primary function of **THE FIBONACCI QUARTERLY** is to serve as a focal point for widespread interest in the Fibonacci and related numbers, especially with respect to new results, research proposals, challenging problems, and innovative proofs of old ideas.

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All back issues of **THE FIBONACCI QUARTERLY** are available in microfilm or hard copy format from **UNIVERSITY MICROFILMS INTERNATIONAL, 300 NORTH ZEEB ROAD, DEPT. P.R., ANN ARBOR, MI 48106.** Reprints can also be purchased from **UMI CLEARING HOUSE** at the same address.

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# *The Fibonacci Quarterly*

*Founded in 1963 by Verner E. Hoggatt, Jr. (1921-1980)  
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THE OFFICIAL JOURNAL OF THE FIBONACCI ASSOCIATION  
DEVOTED TO THE STUDY  
OF INTEGERS WITH SPECIAL PROPERTIES

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