ADVANCED PROBLEMS AND SOLUTIONS

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Send all communications concerning Advanced Problems and Solutions to Verner E. Hoggatt, Jr., Mathematics Department, San Jose State College, San Jose, California. This department especially welcomes problems believed to be new or extending old results. Proposers should submit solutions or other information that will assist the editor. To facilitate their consideration, solutions should be submitted on separate signed sheets within two months after publication of the problems.

H-78 *Proposed by Verner E. Hoggatt, Jr., San Jose State College, San Jose, California*

(i) Show $$\frac{x^{n-1}}{(1-x)^n} = \sum_{m=0}^{\infty} \binom{m}{n-1} x^m, \quad (n \geq 1)$$

where $\binom{m}{n}$ are the binomial coefficients. See "Diagonals of Pascal's Triangle", D. C. Duncan, pg 1115, AMM, Dec. 1965.

(ii) Show $$\frac{x}{1-x-x^2} = \sum_{m=0}^{\infty} \begin{bmatrix} m \\ 1 \end{bmatrix} x^m$$

$$\frac{x^2}{1-2x-2x^2+x^3} = \sum_{m=0}^{\infty} \begin{bmatrix} m \\ 2 \end{bmatrix} x^m ,$$

$$\frac{x^3}{1-3x-6x^2+3x^3+x^4} = \sum_{m=0}^{\infty} \begin{bmatrix} m \\ 3 \end{bmatrix} x^m ,$$

where $\begin{bmatrix} m \\ n \end{bmatrix}$ are the Fibonomial coefficients as in H-63, April 1965, FQJ and H-72 of Dec. 1965, FQJ.

The generalization is:

$$\text{Let} \quad f(x) = \sum_{h=0}^{k} (-1)^{h(h+1)/2} \begin{bmatrix} k \\ h \end{bmatrix} x^h ,$$

then show

$$\frac{x^{k-1}}{f(x)} = \sum_{m=0}^{\infty} \left[{m \atop k-1}\right] x^m , \qquad (k \geq 1) \ .$$

H-79 *Proposed by J.A.H. Hunter, Toronto, Ontario, Canada*

Show

$$F_{n+1}^4 + F_n^4 + F_{n-1}^4 = 2\left[2F_n^2 + (-1)^n\right]^2$$

H-80 *Proposed by J.A.H. Hunter, Toronto, Ontario, Canada and Max Rumney, London, England*

Show

$$\sum_{r=0}^{n} \binom{n}{r} F_{r+2}^2 = \sum_{r=0}^{n} \binom{n-1}{r} F_{2r+5}^2 \ .$$

H-81 *Proposed by Vassili Daiev, Sea Cliff, N.Y.*

Find the nth term of the sequence

1, 1, 3, 1, 5, 3, 7, 1, 9, 5, 11, 3, 13, 7, 15, 1, 17, 9, 19, 5, . . .

H-82 *Proposed by Verner E. Hoggatt, Jr., San Jose State College, San Jose, California*

If $f_0(x) \equiv 0$ and $f_1(x) \equiv 1$, $f_{n+2}(x) = xf_{n+1}(x) + f_n(x)$ then show

$$\mathrm{Tan}^{-1}\frac{1}{x} = \sum_{n=1}^{\infty} \mathrm{Tan}^{-1}\left(\frac{x}{f_{2n+1}(x)}\right) \ .$$

H-83 *Proposed by Mrs. William Squire, Morgantown, West Va.*

Show

$$\sum_{t=1}^{\left[\frac{m+1}{2}\right]} (-1)^{t-1} \binom{m-t}{t-1} 3^{m+1-2t} = F_{2m} ,$$

where $[x]$ is the greatest integer function.

SOLUTIONS

THE LOST IS FOUND

H-42 *Proposed by J.D.E. Konhauser, State College, Pa.*

A set of nine integers having the property that no two pairs have the same sum is the set consisting of the nine consecutive Fibonacci numbers, 1, 2, 3, 5, 8, 13, 21, 34, 55 with total sum 142. Starting with 1, and annexing at each step the smallest positive integer which produces a set with the stated property yields the set 1, 2, 3, 5, 8, 13, 21, 30, 39 with sum 122. Is this the best result? Can a set with lower total sum be found?

Solution by Frank Urbanija, Student, St. Joseph High School, Cleveland, Ohio

The following solution is submitted:

$$1, 2, 3, 5, 7, 15, 20, 25, 41,$$

the sum of which is 121.

Editorial Comment: This is the missing solution and the best received to date.

A BETTER PROBLEM SOLUTION

H-74 *Proposed by Douglas Lind, University of Virginia, Charlottesville, Va.*

Let $f(n)$ denote the number of positive Fibonacci numbers not greater than a specified integer n. Show that for $n > 1$

$$f(n) = \left[K \ln(n\sqrt{5} + \tfrac{1}{2}\right] ,$$

where $[x]$ denotes greatest integer not exceeding x, and K is a constant nearly equal to 2.078086943. (See H. W. Gould's Non-Fibonacci Numbers, Oct. 1965, FQJ).

Comments by John D. Cloud, Manhattan Beach, California

Problem H-74 in the Dec. 1965 issue of the Fibonacci Quarterly is not new. I proposed the same problem in the Nov. 1963 issue of the American Mathematical Monthly (Vol. 70, No. 9, p. 1005, prob. E1636). A more precise solution than the one indicated by Mr. Lind in the FQ appears on p. 798 of the AMM for Sept. 1964 (Vol. 71, No. 7).

The solution by William D. Jackson alluded to shows:

The number of Fibonacci numbers not greater than N is the greatest integer less than

$$\frac{\log\left[(N+\frac{1}{2})\sqrt{5}\right]}{\log\left(\frac{1+\sqrt{5}}{2}\right)} - 1 .$$

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A FIBONACCI CROSSWORD PUZZLE

H.W. Gould
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