EXPLORING SPECIAL FIBONACCI RELATIONS

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In a previous exploration section, readers were introduced to the problem of generalized Fibonacci-Lucas relations. We denoted the terms of the generalized Fibonacci sequence as f_n and those of the associated generalized Lucas sequence as g_n where

$$g_n = f_{n-1} + f_{n+1}$$

Recall also that the sequence (2, 9) means the Fibonacci sequence with $f_1 = 2$ and $f_2 = 9$. The following represent some curious results obtained in trying to express $f_n g_n$ as a linear combination of f's and g's.

Sequence	Formula for $f_n g_n$
(2,9)	$f_n g_n = f_{2n+5} - g_{2n-2}$
(3,7)	$f_n g_n = g_{2n+3} - f_{2n-1}$
(3,10)	$f_n g_n = g_{2n+3} + f_{2n} + g_{2n-1}$
(3,11)	$f_n g_n = g_{2n+4} - g_{2n} - f_{2n-3}$
(4,13)	$f_n g_n = g_{2n+4} + f_{2n-4}$
(5,11)	$f_n g_n = g_{2n+3} + f_{2n+3} + f_{2n-2}$
(5,13)	$f_n g_n = g_{2n+4} + f_{2n-1} + f_{2n-4}$
(6,13)	$f_n g_n = g_{2n+4} + g_{2n-1}$
(6,17)	$f_n g_n = g_{2n+5} - f_{2n+3} + f_{2n-4}$
(7,17)	$f_n g_n = g_{2n+4} + g_{2n+2} + f_{2n-4}$

Now, of course, it must be recognized that these linear expressions could be represented in an infinity of different ways. However, it does not seem that they are all one and the same relation. If not, then we have specific relations that characterize the individual sequences. So the following questions are raised:

(1) Can the above formulas for $f_n g_n$ be unified into one formula?
(2) If not, can other instances be found of this type of phenomenon?
(3) When is it that we have particular formulas for each Fibonacci sequence rather than a general formula for all sequences?

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All subscription correspondence should be addressed to Brother U. Alfred, St. Mary's College, Calif. All checks ($4.00 per year) should be made out to the Fibonacci Association or the Fibonacci Quarterly. Manuscripts intended for publication in the Quarterly should be sent to Verner E. Hoggatt, Jr., Mathematics Department, San Jose State College, San Jose, Calif. All manuscripts should be typed, double-spaced. Drawings should be made the same size as they will appear in the Quarterly, and should be done in India ink on either vellum or bond paper. Authors should keep a copy of the manuscripts sent to the editors.

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