FIBONACCI ON EGYPTIAN FRACTIONS

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When J. J. Sylvester wrote in [1] of his method for expressing any rational number between 0 and 1 as a sum of a finite number of unitary fractions, he was not aware that the method was known to Leonardo Pisano in 1202. B. M. Stewart in [2] has a chapter on Egyptian fractions, but makes no mention of Fibonacci. It is hoped that the following translation will make it possible for Fibonacci to speak for himself. In preparing the text which is to be incorporated in a complete, new edition of the Liber Abacci, we have used pp. 77–83 of Baldassare Boncompagni's Scritti di Leonardo Pisano, Vol. 1 (Rome, 1857). We have consulted microfilms of the manuscripts listed in the bibliography in attempting to correct apparent errors. At the same time, we have tried to reproduce Fibonacci's style as closely as seems consistent with readability.

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In this chapter, we explain how to break up rational numbers into unitary fractions so that you may be able to distinguish more intelligently about fractions having any denominator, when considering what part or parts of a unit integer they are.

This work is divided into 7 sections, the first of which is when the larger number, which is below the bar, is divided by the smaller. The rule for this category is that you divide the larger by the smaller and you will have the part which the smaller is of the larger. E.g., we want to know what part 3/12 is to a unit integer. If 12 is divided by 3, the result is 4, for which you say 1/4, which is 3/12 of a unit integer. In the same way 4/20 is 1/5 of a unit integer; 5/100 is 1/20, since 100 divided by 5 gives 20. You should reason in the same way about similar cases.

Now this category is divided into 3 parts, the first of which is simple, the second compound; the third is called compound reversed. The simple is the one I just mentioned. The compound is when the simple has to do with the parts of a second number, as in the case of

$$\frac{\frac{2}{4}}{9}^{*} ,$$

since 2/4, which belongs to the first simple category, is divided by 9; wherefore,

$$\frac{\frac{2}{4}}{9}$$

becomes

$$\frac{\frac{1}{2}}{9} = \frac{1}{18} ,$$

while

$$\frac{\frac{2}{6}}{9}$$

becomes

$$\frac{\frac{1}{3}}{9}$$

and

$$\frac{\frac{3}{9}}{10}$$

becomes

$$\frac{\frac{1}{3}}{10} ,$$

since 3/9 reduced is 1/3, which compounded with 1/10 produces

$$\frac{\frac{1}{3}}{10} ;$$

and you should reason in this same way about similar cases. The first category compound reversed** is

$$\frac{3/5}{9} ,$$

*This is a 20th-century interpretation of what Leonardo writes as 20/49. We have consistently made such conversions in this translation.
**Notice his awareness of the commutative property!

this being

$$\frac{\frac{3}{9}}{5}$$

which equals

$$\frac{\frac{1}{3}}{5} \qquad ;$$

reason about

$$\frac{\frac{4}{7}}{8}$$

in the same way; it reverses to

$$\frac{\frac{4}{8}}{7} = \frac{\frac{1}{2}}{7} \text{ and } \frac{\frac{5}{9}}{10}$$

becomes

$$\frac{\frac{5}{10}}{9} = \frac{\frac{1}{2}}{9} \qquad .$$

The second category is when the larger number is not divisible by the smaller, but from the smaller can be made such parts that the larger is divided by any of the parts themselves. The rule for this category is that you make parts of the smaller by which the larger can be divided; and let the larger be divided by each one of these parts, and you will have unitary fractions which will be the smaller of the larger. E.g., we want to separate 5/6 into unitary fractions. Since 6 is not divisible by 5, 5/6 cannot be of the first category; but since 5 can be partitioned into 3 and 2, by each of which the larger, namely 6, is divisible, 5/6 is proved to belong to the second category. Hence, when 6 is divided by 3 and 2, the result is 2 and 3; for the 2, one takes 1/2, and for 3, take 1/3. Therefore 5/6 = 1/3 + 1/2 of the unit integer; or otherwise, by separating 5/6 into 3/6 and 2/6, each will be one of those two fractions: 3/6, belonging to the first category, equals 1/2, and 2/6 equals 1/3 of one; and 5/6 equals 1/3 + 1/2, as we said above. Likewise, if you resolve 7/8 into 4/8, 2/8 and 1/8, you will have 1/2 for 4/8 and 1/4 for 2/8 and 1/8 for 1/8, i.e., for

7/8 you will have 1/8 + 1/4 + 1/2. Now this second category has in like manner a compound part and a compound reversed part.

$$\frac{\frac{3}{4}}{10}$$

belongs to the compound part, since 3/4 according to the second category is 1/4 + 1/2; wherefore for

$$\frac{\frac{3}{4}}{10}$$

one gets the compounds

$$\frac{\frac{1}{2}}{10} \text{ and } \frac{\frac{1}{4}}{10} ,$$

i.e., 1/20 and 1/40. Likewise,

$$\frac{\frac{5}{8}}{9} \text{ becomes } \frac{\frac{1}{2}}{9} \text{ and } \frac{\frac{1}{8}}{9} ,$$

since 5/8 = 1/2 + 1/8; but for

$$\frac{\frac{5}{8}}{10} ,$$

since it belongs to the first category reversed, you will not resolve into

$$\frac{\frac{1}{2}}{10} \text{ and } \frac{\frac{1}{8}}{10} ,$$

since by the first category it should be reversed into

$$\frac{\frac{5}{10}}{8} = \frac{\frac{1}{2}}{8} ;$$

and this will take place because of the affinity which 5, which is above the 8, has for 10.* Now regarding the compound reversed part of this category, an

*Recall that in his notation it reads $\frac{5\ \ 0}{8\ 10}$.

example is

$$\frac{\frac{3}{5}}{10}\text{, which is reversed into }\frac{\frac{3}{10}}{5} = \frac{\frac{1}{5}}{5} + \frac{\frac{1}{10}}{5}\text{ ,}$$

i. e., 1/25 + 1/50; because 3/10 is reduced in simple form into 1/5 + 1/10, therefore

$$\frac{\frac{3}{10}}{5}$$

compounded will be resolved into

$$\frac{\frac{1}{5}}{5} \quad\text{and}\quad \frac{\frac{1}{5}}{10} \quad .$$

In like manner

$$\frac{\frac{5}{7}}{8}\ \text{gives}\ \frac{\frac{5}{8}}{7} = \frac{\frac{1}{2}}{7} + \frac{\frac{1}{8}}{7} \quad ;$$

and you should reason in this way in similar cases. But since we recognize that the parts of the first and second category are necessary before the others in computations, we take pains to show at present in certain tables the breakdowns of the parts of certain numbers, which you should be eager to learn by heart in order better to understand what we mean in this section.*

The third category is when one more than the larger number is divisible by the smaller. The rule for this category is that you divide the number which is more than the larger by the smaller, and the quantity resulting from the division will be such part of the unit integer as is the smaller of the larger, plus the same part of the part which one is of the smaller number. E. g., we want to make unitary fractions of 2/11, which is in this category since one more than 11, i. e., 12, is divisible by 2, which is above the bar; since 6 is produced by this division, the results are 1/6 plus 1/6 of 1/11, i. e.,

$$\frac{\frac{1}{6}}{11}$$

as the unitary fractions of 2/11. In the same way, for 3/11 you will have 1/4 and

*To conserve space we have omitted his table of breakdowns for all rational numbers between 0 and 1 having denominators 6, 8, 12, 20, 24, 60, or 100.

$$\frac{\frac{1}{4}}{11}, \quad \text{i.e.,} \quad \frac{1}{44} + \frac{1}{4} .$$

And for 4/11 you will have

$$\frac{1}{3} + \frac{\frac{1}{3}}{11}, \quad \text{i.e.,} \quad \frac{1}{33} + \frac{1}{3} ;$$

and for 6/11 you will have

$$\frac{1}{2} + \frac{\frac{1}{2}}{11}, \quad \text{i.e.,} \quad \frac{1}{22} + \frac{1}{2} ;$$

and for 5/19 you will have

$$\frac{\frac{1}{4}}{19}, \quad \text{i.e.,} \quad \frac{1}{76} + \frac{1}{4} ,$$

since 5 which is above 19 is 1/4 of 20, which is 1 + 19. The third category as well is also compounded twice, as in the case of

$$\frac{\frac{2}{3}}{7} \text{ which is } \frac{\frac{1}{2}}{7} + \frac{\frac{1}{6}}{7}, \text{ since } \frac{2}{3} \text{ is } \frac{1}{6} + \frac{1}{2} ;$$

likewise

$$\frac{\frac{4}{7}}{9} \text{ is } \frac{\frac{1}{2}}{9} + \frac{\frac{1}{14}}{9}, \text{ since } \frac{4}{7} \text{ is } \frac{1}{14} + \frac{1}{2} .$$

And this same category is also reversed, as

$$\frac{\frac{3}{7}}{11} \text{ or } \frac{\frac{3}{7}}{8} ; \text{ for } \frac{\frac{3}{7}}{11} \text{ is } \frac{\frac{3}{11}}{7} .$$

The 3/11 by the third category is 1/44 + 1/4; wherefore

$$\frac{\frac{3}{11}}{7} \text{ is } \frac{\frac{1}{4}}{7} + \frac{\frac{1}{44}}{7} . \text{ Likewise } \frac{\frac{3}{7}}{8} \text{ is reversed to } \frac{\frac{3}{8}}{7},$$

which belongs to two compounded categories, namely the second and third. According to the second compounded category

$$\frac{3/8}{7} \text{ is } \frac{1}{7}\left(\frac{1}{8} + \frac{1}{4}\right) = \frac{1/4}{7} + \frac{1/8}{7} \quad ;^{*}$$

also, according to the third category compounded,

$$\frac{3/8}{7} \text{ is } \frac{1}{7}\left(\frac{1}{24} + \frac{1}{3}\right) ,$$

since 3/8 results in 1/24 + 1/3; and you should reason in the same way about similar cases.

There are times when from this same category two parts can be made from the smaller number above the bar, by either of which one more than the larger is divided without remainder, as in 8/11 and 9/11.** For two parts can be made from 8/11, namely 6/11 and 2/11; whence for 6/11 we have, according to this reasoning, two unitary fractions, namely 1/22 + 1/2, and for 2/11 we have 1/66 + 1/6. Therefore for 8/11 we have 1/66 + 1/22 + 1/6 + 1/2. Likewise, for 9/11, through its being resolved into 6/11 and 3/11, we have

$$\frac{1}{44} + \frac{1}{22} + \frac{1}{4} + \frac{1}{2} \quad ;$$

and for 10/11 we have

$$\frac{1}{33} + \frac{1}{22} + \frac{1}{3} + \frac{1}{2} \quad ,$$

since the 10 above the 11 is 1/3 + 1/2 of 12, which is one more than the 11 under the bar.

The fourth category is when the larger is prime and one more than the larger is divided by one less than the smaller, as 5/11 and 7/11. The rule for this category is that you subtract one from the smaller, from which you will make one unitary fraction such as the number will be which is under the bar;

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*Note the distributive law! His notation is $\dfrac{1\ \ 1\ \ 0}{8\ \ 4\ \ 7}$.

**It appears that he is trying to generalize. If so, he must have recognized a flaw; for in the next section he uses a different approach and then you will have as remainders parts of the third category, as in the case when you subtract 1/11 from 5/11, with the remainder being 4/11. For this, by the third category, you will have unitary fractions 1/33 + 1/3; with the above-written 1/11 added on, the result is

$$\frac{1}{33} + \frac{1}{11} + \frac{1}{3} \quad .$$

In the same way for 7/11 you will have

$$\frac{1}{22} + \frac{1}{11} + \frac{1}{2} \; ;$$

and for 3/7 you will have

$$\frac{1}{28} + \frac{1}{7} + \frac{1}{4} \quad ;$$

and for 6/19 you will have

$$\frac{1}{76} + \frac{1}{19} + \frac{1}{4} \quad ;$$

and for 7/29 you will have

$$\frac{1\frac{1}{5}}{29} + \frac{1}{5} \quad , \quad \text{i.e.,} \quad \frac{1}{145} + \frac{1}{29} + \frac{1}{5} \quad .$$

The fifth category is when the larger number is even, and one more than the larger is divisible by two less than the smaller. The rule for this category is that you subtract 2 from the smaller number, and this 2 will belong to the first category, but the remainder will belong to the third, as in 11/26 from which if you subtract 2/26, which equals 1/13 according to the rule of the first category, the remainder is 9/26, which equals

$$\frac{1/3}{26} + \frac{1}{3} \quad , \quad \text{e.g.,} \quad \frac{1}{78} + \frac{1}{3} \quad .$$

To this add 1/13; the result will be

$$\frac{1}{78} + \frac{1}{13} + \frac{1}{3}$$

as the unitary fractions of 11/26; and in the same way for 11/62 you will have

$$\frac{1/7}{62} + \frac{1}{31} + \frac{1}{7} \quad .$$

The sixth category is when the larger number is divisible by 3, and one more than the larger is divisible by 3 less than the smaller, as in 17/27. The rule for this is that from the parts themselves you will subtract three parts, i. e., that you will subtract 3 from the lesser. These three parts will belong to the first category, the rest to the third, as when from 17/27 you subtract 3/27 (1/9 according to the first category of our topic); and to 14/27, which by the third category is 1/54 + 1/2, adding the 1/9 written above, you get

$$\frac{1}{54} + \frac{1}{9} + \frac{1}{2}$$

as the parts of 17/27. In the same way for 20/33 you will have

$$\frac{1}{66} + \frac{1}{11} + \frac{1}{2} \quad .$$

The seventh category is when none of the above-described categories occurs. The rule for this category is very useful, since through it the parts of certain above-described categories, viz., the second, fourth, fifth and sixth categories, are occasionally found better than through the rules of those categories themselves. Hence the parts of the four categories themselves are always to be derived through this seventh rule, so that you can more precisely discover the neater parts, either through the rules of the categories themselves or through the present rule. The rule for this category is that you divide the larger number by the smaller; and when the division itself is not even, look to see between what two numbers that division falls. Then take the larger part,* and subtract it, and keep the remainder. If this will be from one of the above-described categories, then you will take the larger part of the remainder itself; and you will do this until the parts of one of the above-described categories remain, or until you have all unitary fractions. E. g., we want to break down 4/13 into unitary fractions. Now 13 divided by 4 falls between 3 and 4; wherefore 4/13 of the unit integer is less than 1/3 and more than 1/4 of it; wherefore

*By larger part, he evidently means the unitary fraction with the larger denominator. In the example this would be 1/4.

we perceive that 1/4 is the largest unitary fraction that can be taken from 4/13. For 13/13 makes the unit integer; wherefore one fourth of 13/13, i. e.,

$$\frac{13}{4 \times 13}$$

is 1/4 of the unit integer; wherefore subtract 13/(4 x 13) from 4/13, and the remainder will be 3/(4 x 13), which by the second category is

$$\frac{1}{13}\left(\frac{1}{4} + \frac{1}{2}\right), \text{ i. e., } \frac{1}{52} + \frac{1}{26} .$$

Or since 3/(4 x 13) is 3/52, which by the rule for the second category is in like manner 1/52 + 1/26, we thus have for 4/13 three unitary fractions, namely

$$\frac{1}{52} + \frac{1}{26} + \frac{1}{4} .$$

Otherwise, you can find the parts of 3/52 by this seventh rule, namely by dividing by 3; the result is 17, and more; wherefore 1/18 is the largest part which is in 3/52. Consequently, 52 divided by 18 gives $2 + \frac{8}{9}$; if this is taken from 3, the remainder is

$$\frac{1}{9 \times 52} \text{ or } \frac{1}{468} .$$

Therefore for 3/52 we have

$$\frac{1}{468} + \frac{1}{18} ;$$

wherefore for 4/13 we have

$$\frac{1}{468} + \frac{1}{18} + \frac{1}{4} .$$

Likewise, you will make unitary fractions of 9/61 in the following way. Divide 61 by 9, and the result will be 6, and more; wherefore you will have 1/7 as the largest unitary fraction of 9/61. And so you will divide 61 by 7, with the result $8 + \frac{5}{7}$, which are sixty-first parts of one; subtract this from 9/61, and the remainder will be

$$\frac{2}{7 \text{ x } 61}, \text{ i. e., } \frac{2}{427}; \text{ this } \frac{2}{427} \text{ is } \frac{1}{214} \text{ and } \frac{1}{214 \text{ x } 427},$$

according to the third category; therefore 9/61 is

$$\frac{1}{214 \text{ x } 427} + \frac{1}{214} + \frac{1}{7} .$$

Since 2/(7 x 61) by the compound third category results in

$$\frac{1}{4 \text{ x } 61} \text{ and } \frac{1}{28 \text{ x } 61},$$

consequently for 9/61, the result is

$$\frac{1}{1708} + \frac{1}{244} + \frac{1}{7} .*$$

Likewise, we want to demonstrate this same method concerning 17/29. If 29 is divided by 17, we obtain 1, and more; wherefore we perceive that 17/29 is more than half of the unit integer; and it is to be noted that 3/3, 4/4, 5/5, or 6/6 makes the unit integer; in like manner 29/29 makes the unit integer. If we take half of it, i. e., $14\frac{1}{2}/29$, and take this from 17/29, the remainder will be $2\frac{1}{2}/29$, i. e., 5/58; wherefore 17/29 is 5/58 + 1/2. Of this 5/58 one must make unitary fractions, namely by this same category; wherefore divide 58 by 5, and the result will be 11, and more. Whence one perceives that 1/12 is the largest unitary fraction that is in 5/58; whence one should take 1/12 from 58/58, i. e., from the integer, with the result $4\frac{5}{6}/58$, which is less than 5/58 by 1/(6 x 58), i. e., 1/348; and so you will have for 17/29 three unitary fractions, namely

$$\frac{1}{348} + \frac{1}{12} + \frac{1}{2} .$$

*By compound third category he means factor out 1/61 from 2/(7 x 61) and apply the third category method to 2/7. This gives 2/7 = 1/28 + 1/4, and so 2/(7 x 61) = 1/61 (1/28 + 1/4) = 1/(61 x 28) + 1/(61 x 4).

Now there is in similar cases a certain other universal rule, namely, that you find a number which has in it many divisors like 12, 24, 36, 48, 60, or any other number which is greater than half the number under the bar, or less than double it, so that for the above-written 17/29 we take 24, which is more than half of 29; and therefore multiply the 17 which is above the bar by 24, and it will be 408; divide this by 29 and by 24, with the result

$$\frac{14\frac{2}{29}}{24} .$$

Then see what part 14 is of 24; the result is

$$\frac{1}{4} + \frac{1}{3} \text{ or } \frac{1}{12} + \frac{1}{2} \quad ;$$

keep these as parts of 17/29; and again see what part the 2 which is above the 29 is of 24; the result is 1/12 of it, for which you will have (1/12)/29 among the parts of 17/29. Since 2/29 of 1/24 = 2/24 of 1/29, which is namely 1/348, therefore for 17/29 you will have

$$\frac{1}{348} + \frac{1}{4} + \frac{1}{3}, \text{ or } \frac{1}{348} + \frac{1}{12} + \frac{1}{2} ,$$

as we found out above.

Likewise, if you multiply the 17 which is above the 29 by 36, just as you multiplied it by 24, and divide in a similar way by 29 and by 36, the result is

$$\frac{21\frac{3}{29}}{36}, \text{ the 21 being } \quad \frac{1}{4} + \frac{1}{3} \quad \text{or} \quad \frac{1}{12} + \frac{1}{2} \text{ of } 36;$$

and 3, which is above 29, is 1/12 of 36; and from the 3 which is above 29, the result will be

$$\frac{\frac{1}{12}}{29}, \text{ i.e., } \frac{1}{348} \quad ;$$

and so you will have again as the unitary fractions of 17/29,

$$\frac{1}{348} + \frac{1}{4} + \frac{1}{3}, \quad \text{or} \quad \frac{1}{348} + \frac{1}{12} + \frac{1}{2} .$$

And if you want to know why we multiplied by 24 that 17 which is above the 29, and divided the product by 29, you should know that of 17/29 we made twenty fourths because 24 is a number composed of many numbers, whence its parts derive from the first and second category. For 17/29 has been found, as previously said, to be

$$\frac{14\,\frac{2}{29}}{24}, \text{ of which } \frac{14}{24} \quad ,$$

which is at the head of the bar,* is expressed by the second category as

$$\frac{1}{4} + \frac{1}{3} \text{ or } \frac{1}{12} + \frac{1}{2} \quad ;$$

and for the remainder $\frac{2}{29}/24$, it is expressed by the first category reversed as

$$\frac{\frac{2}{24}}{29}\ , \text{ i.e., } \frac{\frac{1}{12}}{29} \quad .$$

Likewise, when you have multiplied 17 by 36 and divided by 29, then you have made thirty sixths of 17/29. For 29/29 is equal to 36/36; wherefore the ratio which 29 is to 36 will also hold true for 17 to the fourth number; wherefore we multiplied the third number, namely 17, by the second, namely 36, and divided the product by the first, because when the four numbers are proportional, the multiplication of the second by the third is equal to the multiplication of the first by the fourth, as has been shown by Euclid.

Likewise, if you want to break down 19/53 into unitary fractions, although it belongs to the fourth category, when one more than 53 is divided by one less than 19, whence for 19/53 you will have

$$\frac{1}{159} + \frac{1}{53} + \frac{1}{3} \quad ,$$

let us next show in what way it should be done by the seventh category. Now 53 divided by 19 falls between 2 and 3; wherefore we have 1/3 as the largest unitary

*In his notation it reads $\frac{2}{29}\frac{14}{24}$. Since he reads numbers from right to left, $\frac{14}{24}$ is at the beginning or head of the fraction.

fraction which can be taken from 19/53; and subtract a third of 53, namely $17\frac{2}{3}$, from 19, with the remainder being $1\frac{1}{3}$, i.e., $1\frac{1}{3}/53$; therefore the unitary fractions of 19/53 are

$$\frac{1}{159} + \frac{1}{53} + \frac{1}{3} \quad ,$$

as we discovered through the rule of the fourth category.

Now by this rule one cannot so easily make the unitary fractions of 20/53. Hence you will find them through another rule, viz., by multiplying 20 by some number which has many divisors, as we have said previously. Now if 20 is multiplied by 48 and divided by 53 and then by 48, the result is

$$\frac{18\frac{6}{53}}{48}\ , \quad \text{the 18 being } \frac{1}{8} + \frac{1}{4} \text{ of } 48, \quad \text{or} \quad \frac{1}{24} + \frac{1}{3} \quad ;$$

and the 6 which is above the 53 is 1/8 of 48; wherefore it will be $\frac{1}{8}/53$, since the 6 is above the 53; therefore for the unitary fractions of 20/53 you have

$$\frac{\frac{1}{8}}{53} + \frac{1}{24} + \frac{1}{3} \quad ;$$

and you should strive to operate in this way in all similar cases; and when you cannot have through any of the above-mentioned rules convenient unitary fractions of any similar cases, you should strive to find them through one of the others; and one should note that there are many fractions which should be adapted before they are broken down into unitary fractions, namely when the larger number is not divided by the smaller and has in turn some common divisor, as in the case of 6/9, each of which numbers is exactly divided by 3; wherefore you will divide each of them by 3, with the resulting 2 above the bar and 3 below it, i.e., 2/3, which belongs to the third category, when one more than 3 is divisible by 2, whereby they are 1/6 + 1/2. It is the same in the case of 6/8, each of which numbers is divisible by 2, whence they are reduced to 3/4, equal to 1/4 + 1/2 by the second category; and you should reason in this way about similar cases. And if several fractions are under one bar, they should be reduced to one fraction beneath the bar, as in the case of $3\frac{1}{2}/8$, which is 7/16. And they are reduced as follows: 3, which is above the 8, is multiplied by 2, and 1 is added; we put it down, and thus we have 7; and you will multiply 2 by 8, which is under the bar, and get 16; this 16 we put under the bar, and above it we put the 7.

Likewise, $\dfrac{4\,\frac{3\frac{2}{3}}{5}}{9}$ * is $\dfrac{71}{135}$,

which is found according to the above-written method, namely by multiplying the 4 above the 9 by 5 and adding 3; this product is multiplied by 3 and 2 is added; and so we have 71 above the bar; and from multiplying 3 by 5 and that product by 9 we have 135 below the bar; this 71/135 according to the seventh rule is broken down into

$$\frac{1}{270} + \frac{1}{45} + \frac{1}{2} .$$

And it should be noted that when by the seventh rule you take the largest part, which the smaller number will be of the larger, and leave the unitary fractions that remain, the result is less than elegant. You will leave that larger part and work with the other following part, which is less than it; so that if the larger part is 1/5, you will work with 1/6; and if it is 1/7, you will work with 1/8. E.g., in 4/49 the largest part is 1/13; when this is taken from 4/49, the remainder is

$\dfrac{\frac{3}{13}}{49}$, namely $\dfrac{3}{637}$, which by the fourth category is $\dfrac{\frac{1}{319}}{637} + \dfrac{1}{319}$;

therefore, for 4/49 we have

$$\frac{\frac{1}{319}}{637} + \frac{1}{319} + \frac{1}{13} ,$$

which is less than elegant; wherefore you will abandon 1/13 and work with 1/14; when this is taken from 4/49, the remainder is

$\dfrac{\frac{1}{2}}{49}$, i.e., $\dfrac{1}{98}$; and so for $\dfrac{4}{49}$ we have $\dfrac{1}{98} + \dfrac{1}{14}$,

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*In his notation $\dfrac{2\ 3\ 4}{3\ 5\ 9}$.

which are more elegant fractions than those previously found. They are found also in another way, namely by dividing the 4 above the 49 by a divisor of 49, with the result $\frac{4}{7}/7$, which by the third category compounded is

$$\frac{1}{7}\left(\frac{1}{14} + \frac{1}{2}\right); \quad \text{for } \frac{\frac{1}{2}}{7} \quad \text{is} \quad \frac{1}{14}, \text{ and } \frac{\frac{1}{14}}{7} \quad \text{is} \quad \frac{1}{98};$$

and thus for 4/49 we have in like manner 1/98 + 1/14.

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1. J. J. Sylvester, "On a Point in the Theory of Vulgar Fractions," American Journal of Mathematics, III (1880), pp. 332-335, 388-389.

2. B. M. Stewart, Theory of Numbers, 2nd ed., Macmillan, New York, 1964.

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CORRECTION

In "An Almost Linear Recurrence," by Donald E. Knuth, April 1966 Fibonacci Quarterly, p. 123, replace c_n by $\ln c_n$ in Eq. 13.

In "On the Quadratic Character of the Fibonacci Root," by Emma Lehner, April 1966 Fibonacci Quarterly, please make the following corrections:

p. 136, line -9: This line should end in θ.

p. 136, line -5: Replace 5 by $\theta\sqrt{5}$.

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